Performance Analysis of Transmit Beamforming under Employment of Generalized Receiver in Wireless Communication Systems

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Abstract— The bit error rate (BER) performance analysis of transmit beamforming technique employing the generalized receiver (GR) under the binary phase shift keying (BPSK) modulation over Rayleigh fading channel is investigated. The scheme uses two transmit antennas and one receive antenna. Simulation result demonstrates a high performance gain under employment of GR with transmit beamforming technique in comparison with the neyman pearson (NP) receiver under the same conditions.

Keywords— Generalized receiver (GR), rayleigh fading channel, bit error rate (BER), transmit beamforming, and transmit diversity.

I. INTRODUCTION

In urban and indoor environments, the line-of-sight between the transmitter and receiver has a random character and the transmitted signal is reflected along multiple paths. This can introduce phase shifts, time delays, attenuations, and distortions that can destructively interfere with one another at the aperture of the receiving antenna. Antenna diversity is effective solution to mitigate the multipath fading [1]. Multiple antennas offer to the receiver several observations of the same signal and each antenna experiences a different interference source. Thus, if one antenna experiences a deep fade, it is likely that other antennas may have sufficient radio channel conditions. Multiple antenna system can provide a robust radio channel transmission. The experimental results of dual and triple diversity are discussed in [2]. Antenna diversity is a practical, effective, and hence, widely applied technique to reduce the effect of multipath fading. In a multiple input and single output (MISO) system, if the channel state information (CSI) is available at the transmitter, the diversity and array gains with transmit beamforming can be achieved via maximal ratio transmission (MRT) [3]. The efficient use of CSI for transmit beamforming is discussed in [4]. The performance analysis of MRT with maximal ratio combining for MIMO channel is presented in [5]. Theoretical analysis of transmit beamforming with imperfect feedback is delivered in [6] and [7]. The effect of feedback delay and feedback errors on the received signal-to-noise ratio (SNR) is investigated in [8]. On the other hand, the bit error probability (BEP) degradation due to feedback errors with selection and co-phasing feedback schemes are investigated in [9].

Moreover, the design and analysis of transmit beamforming under finite-rate constraints are discussed in [10]-[13].

In this paper, we employ the generalized receiver (GR) constructed based on the generalized approach to signal processing (GASP) in noise [14], [15]. The idea to employ GR in multiple antenna system is mentioned in [16]. The performance comparison is made between GR and neymanpearson (NP) receiver for MIMO system using BPSK modulation over Rayleigh fading channel with different equalization techniques. Using GR for multiple antenna system leads us to low bit error rate (BER) in comparison with the NP receiver under the same conditions. The GR performance with quadrature subbranch HS/MRC and HS/MRC schemes for a 1-D signal modulation in Rayleigh fading is investigated in [17]. In this paper, we study the transmit beamforming technique employing GR under BPSK modulation over Rayleigh fading channel with two transmit antennas and one receive antenna. We assume that the required CSI for GR and NP receiver are obtained. The BER performance of transmit beamforming technique over Rayleigh fading channel employing GR has a great advantage in comparison with the receiver based on the NP criterion.

The rest of this paper is organized as follows. In section 2 we describe the GR. Transmit beamforming with two transmit antennas and one receive antenna is discussed in section 3. Simulation results are given in section 4 and some conclusions are made in section 5.

II. GENERALIZED RECEIVER

The GR can be simply presented in a form of block diagram shown in Fig.1 [17]. In this flowchart, MSG is the model signal generator (local oscillator), PF is the preliminary filter with the impulse response $h_{PF}(t)$, and AF is the additional filter with the impulse response $h_{AF}(t)$. A resonant frequency of the AF is detuned relative to a resonant frequency of PF on such a value that at the PF output both the signal and noise can be appeared whereas the only noise is appeared at the AF output. A value of detuning between the AF and PF resonant frequencies is more than $4 \div 5\Delta f_a$, where, Δf_a is the signal bandwidth. In this case, the



Figure 1. Principal GR flowchart.

coefficient of correlation is not more than 0.5. If the Gaussian noise n(t) comes in at the AF and PF inputs (the GR linear system front end), the noise forming at the AF and PF outputs is Gaussian, too, because the AF and PF are the linear systems and, in a general case, takes the following form [15] and [17],

$$\xi_{PF}(t) = \sum_{i=-\infty}^{\infty} h_{PF}(t) n(t_i - \tau), \qquad (1)$$

$$\xi_{AF}(t) = \sum_{i=-\infty}^{\infty} h_{AF}(t) n(t_i - \tau).$$
⁽²⁾

If the additive white Gaussian noise (AWGN) with zero mean and two sided power spectral density $N_0/2$ is coming in at the AF and PF inputs (the GR linear system front end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by,

$$\sigma_n^2 = \frac{N_0 \omega_0}{8\Delta_F},\tag{3}$$

where Δ_F is the AF (or PF) bandwidth and ω_0 is the resonance frequency.

The stochastic process at the output of the PF takes the following form [15]:

$$X_i(t) = \sqrt{E_b} a_i(t) + \xi_i(t) \quad i \in [1, N]; \quad 0 \le t \le T ,$$

$$\tag{4}$$

where $\sqrt{E_b}a_i(t)$ is the received signal at the output of the PF; E_b is the signal energy per bit; N is the sample size; [0,T] is the time interval within the limits of which the input stochastic process is observed; $\xi_i(t)$ is the Gaussian noise with zero mean and the spectral power density $\frac{N_0}{2}$ at the output of the PF. The main functioning condition of GR takes the following form:

$$\sqrt{E_b^*}a_i^*(t) = \sqrt{E_b}a_i(t) , \qquad (5)$$

Here $a_i^*(t)$ is the model signal; E_b^* is the model signal energy per bit. The process at the GR output takes the following form:

$$Z_{g}^{out}(t) = 2\sum_{i=1}^{N} \sqrt{E_{b}^{*}} a_{i}^{*}(t)$$

$$Z_{g}^{out}(t) = 2\sqrt{E_{b}^{*}} E_{b} \sum_{i=1}^{N} a_{i}^{*}(t) a_{i}(t) - E_{b} \sum_{i=1}^{N} a_{i}^{2}(t) + 2\sqrt{E_{b}^{*}} \sum_{i=1}^{N} a_{i}^{*}(t) \xi_{i}(t)$$

$$-2\sqrt{E_{b}} \sum_{i=1}^{N} a_{i}(t) \xi_{i}(t) - \sum_{i=1}^{N} \xi_{i}^{2}(t) + \sum_{i=1}^{N} \eta_{i}^{2}(t)$$
(6)

where $\eta_i(t)$ is the noise at the output of the AF. Satisfying the main GR functioning condition given by (5) we obtain,

$$Z_g^{out}(t) = E_b \sum_{i=1}^N a_i^2(t) + \sum_{i=1}^N \eta_i^2(t) - \sum_{i=1}^N \xi_i^2(t)$$
(7)

III. TRANSMIT BEAMFORMING PRINCIPLES

In this paper, we consider two transmit antennas and one receive antenna over Rayleigh fading channel shown in Fig. 2. For the receive antenna, each transmitted signal is multiplied by a random channel gain h_i with i = 1, 2. As considered channel is a Rayleigh fading channel, the real and imaginary parts of h_i are Gaussian distributed with the zero mean and variance equal to 0.5. The complex channel gain between the first transmit antenna and receive antenna is denoted by h_1 and between the second transmit antenna and receive antenna is denoted by h_2 . If the channel phase is denoted by θ_i then the channel gains take the following form [19]:

$$h_{1} = |h_{1}| e^{j\theta_{1}} , \qquad (8)$$

$$h_2 = \left| h_2 \right| e^{j\theta_2} \,, \tag{9}$$

After addition of noise at the receiver, the resulting received signals take the following form:

$$y_1 = h_1 x + n \tag{10}$$

$$y_2 = h_2 x + n \,, \tag{11}$$

where y_1 is the received signal from the first transmit antenna, y_2 is the received signal from the second transmit antenna, x is the transmitted signal, and n is the received noise.

At the receive antenna output the signal can be represented as follows:

$$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x \\ x \end{bmatrix} + n = (h_1 + h_2)x + n, \qquad (12)$$

where y is the received signal combining y_1 and y_2 . When transmit beamforming is applied, each transmitted signal is multiplied by a complex channel gain corresponding to the inverse of the channel phase to ensure that all the transmitted signals are added constructively at the receiver. In this case, the received signal takes the following form:

$$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} e^{-j\theta_1} \\ e^{-j\theta_2} \end{bmatrix} x + n .$$
 (13)

Finally, the received signal can be defined as follows:

$$y = (|h_1| + |h_2|)x + n.$$
(14)

The instantaneous SNR per bit of the received signal takes the following form [19], [20]:

$$\gamma = \left|h\right|^2 \frac{E_b}{N_0},\tag{15}$$

where $\mathbf{h} = \begin{bmatrix} h_2 & h_2 \end{bmatrix}$ is the vector of complex channel gain.

The probability density function (pdf) of γ for Rayleigh fading channel can be defined as follows [19], [20]:

$$p(\gamma) = \frac{1}{\left(E_{_{b}} / N_{_{0}}\right)} e^{\frac{-\gamma}{\left(E_{_{b}} / N_{_{0}}\right)}}.$$
 (16)

The average BER takes the following form [19], [20]:

$$P_{b} = \int_{0}^{\infty} \frac{1}{2} \operatorname{erfc}(\gamma) p(\gamma) d\gamma$$
$$= 0.5 \left[1 - \sqrt{\frac{E_{b} / N_{0}}{(E_{b} / N_{0}) + 1}} \right]. \tag{17}$$



Figure 2. Two branch diversity scheme with one receiver.

IV.SIMULATION

The BER performance comparison between GR and NP receiver under the BPSK modulation over Rayleigh fading channel with transmit diversity and transmit beamforming techniques are presented in Fig. 3. For example, in the case of transmit diversity technique, at $E_b / N_0 = 10 \text{ dB}$; the achieved *BER* for GR is equal to 9.611×10^{-3} , whereas, in the case of NP receiver, at the same E_b / N_0 , the *BER* is equal to 23.22×10^{-3} . We observe that the performance improvement is achieved for both the transmit diversity and transmit beamforming techniques employing GR in comparison with the NP receiver. Under transmit beamforming case, when E_{h}/N_{0} is less than 6 dB, the performance of NP receiver is better than the GR and when E_h / N_0 is greater than 6 dB, GR shows better performance than NP receiver. For example, at $BER = 10^{-4}$, the required energy per bit to noise ratio (E_h/N_0) is equal to 12 dB for GR, whereas, the required energy per bit to noise ratio (E_b / N_0) is equal to 17 dB for NP receiver. Moreover, we achieve 5 dB E_h / N_0 gain at $BER = 10^{-4}$ employing GR in comparison with NP receiver with transmit beamforming technique.

V. CONCLUSION

In this paper the GR BER performance under BPSK modulation over Rayleigh fading channel in wireless communication system with transmit beamforming technique for 2×1 antenna configuration is discussed. The performance comparison between GR and NP receiver with transmit diversity and transmit beamforming techniques demonstrates

the low BER by the employment of GR. Finally, we can see that, GR overcomes the NP receiver performance in both transmit diversity and transmit beamforming cases.



BER for BPSK modulation in Rayleigh channel

Figure 3.BER performance comparison between GR and NP receiver for transmit diversity and transmit beamforming schemes.

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