

Generalized Receiver under Blind Multiuser Detection in Wireless Communications

Vyacheslav Tuzlukov

School of Electrical Engineering & Computer Science, Kyungpook National University, Daegu, South Korea

Tuzlukov@ee.knu.ac.kr

Abstract—A multiuser detection scheme based on the generalized approach to signal processing in noise is proposed. It is shown that under this scheme, the generalized receiver can be designed blindly, i.e., it can be estimated from the received signal with the prior knowledge of only the signature waveform and timing of the user of interest. A blind adaptive implementation based on a signal subspace-tracking algorithm is also developed. It is seen that compared with the minimum-output-energy blind adaptive multiuser detector, the proposed subspace-based blind adaptive generalized receiver offers better performance but higher computational complexity.

I. INTRODUCTION

Direct-sequence spread-spectrum code-division multiple access (DS-SS/CDMA) modulation techniques are considered as a popular multi-access technology for personal, cellular, and satellite communication services [1], [2]. The capacity of CDMA wireless communication systems can be substantially increased under implementation of multiuser detection technique. At the present time, a large amount of research has addressed various multiuser detection schemes [3]. Adaptive multiuser detection has been paid a considerable recent attention [4]. Thus, the methods for adapting the decorrelating, or zero-forcing, linear detector requiring the transmission of training sequences during adaptation have been proposed in [5]-[7]. The minimum-mean-square-error (MMSE) detector, the alternative detector, can be adapted either through the use of training sequences [8]-[11], or in the blind mode, i.e., with the *prior* knowledge of only the signature waveform and timing of the user of interest [12], [13]. Blind adaptation schemes are especially attractive for the downlinks of CDMA systems, since in a dynamic environment, it is very difficult for a mobile user to obtain accurate information on other active users in the channel, such as their signature waveforms; and the frequent use of training sequence is certainly a waste of channel bandwidth.

In this paper, we propose a blind multiuser generalized receiver (GR) constructed according to the generalized approach to signal processing in noise [14]-[18], which is based on signal subspace estimation. Subspace-based high-resolution methods play an important role in sensor array processing, spectrum analysis, and general parameter estimation [19]. Several recent works have addressed the use of sub-spaced-based me-

thods for delay estimation [20], [21] and channel estimation [20], [22] in CDMA systems.

The contribution of this paper is the following. First of all, we show that based on signal subspace estimation, GR can be obtained blindly, i.e., it can be estimated from the received signal with the *prior* knowledge of only the signature waveform and timing of the user of interest. The consistency and the asymptotic variance of the estimations of GR are examined. A blind adaptive implementation based on a signal subspace tracking algorithm is also developed. It is seen the compared, for example, with minimum-output-energy (MOE) blind adaptive detector [12], the blind adaptive GR offers better performance in terms of the steady-state signal-to-interference-noise ratio (SINR).

II. SIGNAL MODEL

Consider a baseband digital direct sequence (DS) CDMA network of K users. The received signal can be modeled as

$$x(t) = a(t) + n(t) , \quad (1)$$

where $n(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2 , and $a(t)$ is the superposition of the data signals of the K users, given by

$$a(t) = \sum_{k=1}^K S_k \sum_{i=-M}^M b_k(i) \alpha_k(t - iT - \tau_k) , \quad (2)$$

where $2M+1$ is the number of data symbols per user per frame T is the symbol interval; S_k denotes the received amplitude; τ_k is the delay; $\{b_k(i)\}$ is the symbol stream and $i = 0, \pm 1, \dots, \pm M$; $\{\alpha_k(t)\}$ is the normalized signaling waveform of the k -th user and $0 \leq t \leq T$. We assume that $\alpha_k(t)$ are supported only within the limits of the interval $[0, T]$ and have unit energy, and that $\{b_k(i)\}$ is a collection of independent equiprobable ± 1 random variables. For the direct-sequence spread-spectrum (DS-SS) multiple-access format, the user signaling waveforms are of the form

$$\alpha_k(t) = \sum_{j=0}^{N-1} \beta_j^k \psi(t - jT_c) , \quad (3)$$

where N is the processing gain; $(\beta_0^k, \beta_1^k, \dots, \beta_{N-1}^k)$ is a signature sequence of ± 1 's assigned to the k -th user; and $\psi(t)$ is a normalized chip waveform of duration T_c , where

$$NT_c = T . \quad (4)$$

In this paper, we restrict our attention to the synchronous case of model given by (2), in which

$$\tau_1 = \tau_2 = \dots = \tau_K = 0 . \quad (5)$$

It is then sufficient to consider the received signal during one symbol interval, and the received signal model becomes

$$x(t) = \sum_{k=1}^K S_k b_k \alpha_k(t) + n(t) , \quad t \in [0, T] . \quad (6)$$

One simple suboptimal way to treat the asynchronous system is the ‘‘one-shot’’ approach, in which a particular transmitted data bit is estimated based on only the received signal within the symbol interval corresponding to that data bit. An asynchronous system of K users can then be viewed as equivalent to a synchronous system with $2K - 1$ users [9], and the results of this paper thus apply in this context as well. Alternatively, an asynchronous CDMA system is a special case of the more general dispersive CDMA system in which the channel introduces the intersymbol interference (ISI), in addition to the multiple-access interference (MAI). The subspace-based techniques considered in this paper can also be extended to such a dispersive CDMA system for blind joint suppression of both MAI and ISI [23].

Consider the synchronous model given by (6). At the receiver, chip-matched filtering followed by chip rate sampling yields an N -vector of chip-matched filter output samples within a symbol interval T

$$\mathbf{x} = \sum_{k=1}^K S_k b_k \mathbf{a}_k + \mathbf{n} , \quad (7)$$

where

$$\mathbf{a}_k = \frac{1}{\sqrt{N}} (\beta_0^k, \beta_1^k, \dots, \beta_{N-1}^k) \quad (8)$$

is the normalized signature waveform vector of the k -th user, and \mathbf{n} is the AWGN vector with $\mathbf{0}$ mean and covariance matrix

$$\mathbf{K}_n = \sigma_n^2 \mathbf{I}_N , \quad (9)$$

where \mathbf{I}_N is the $N \times N$ identity matrix. Thus, we can restrict attention to the discrete-time model given by (7).

III. SUBSPACE-BASED BLIND LINEAR MULTIUSER GR

A. Subspace Concept

For convenience and without loss of generality, we assume that the signature waveforms $\{\mathbf{a}_k\}_{k=1}^K$ of the K users are linearly independent. Denote

$$\mathbf{A} = [\mathbf{a}_1 \mathbf{a}_2 \dots \mathbf{a}_K] \quad (10)$$

and

$$\mathbf{S} = \text{diag}(S_1^2, \dots, S_K^2) . \quad (11)$$

The autocorrelation matrix of the received signal \mathbf{x} takes the following form

$$\mathbf{Q} = E\{\mathbf{r} \mathbf{r}^T\} = \sum_{k=1}^K S_k^2 \mathbf{a}_k \mathbf{a}_k^T + \sigma_n^2 \mathbf{I}_N = \mathbf{A} \mathbf{S} \mathbf{A}^T + \sigma_n^2 \mathbf{I}_N , \quad (12)$$

where $E\{\cdot\}$ is the mathematical expectation and subscript T denotes the transposed matrix. By performing an eigendecomposition of the matrix \mathbf{Q} , we get

$$\mathbf{Q} = \mathbf{V} \mathbf{W} \mathbf{V}^T = [\mathbf{V}_\alpha \ \mathbf{V}_n] \times \begin{bmatrix} \mathbf{W}_\alpha & \\ & \mathbf{W}_n \end{bmatrix} \times \begin{bmatrix} \mathbf{V}_\alpha^T \\ \mathbf{V}_n^T \end{bmatrix} , \quad (13)$$

where

$$\mathbf{V} = [\mathbf{V}_\alpha \ \mathbf{V}_n] ; \quad (14)$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_\alpha & \\ & \mathbf{W}_n \end{bmatrix} ; \quad (15)$$

$$\mathbf{W}_\alpha = \text{diag}(w_1, \dots, w_K) \quad (16)$$

contains the K largest eigenvalues of \mathbf{Q} in descending order and

$$\mathbf{V}_\alpha = [\mathbf{v}_1 \dots \mathbf{v}_K] , \quad (17)$$

contains the corresponding orthonormal eigenvectors;

$$\mathbf{W}_n = \sigma_n^2 \mathbf{I}_{N-K} \quad (18)$$

and

$$\mathbf{V}_n = [\mathbf{v}_{K+1} \dots \mathbf{v}_N] \quad (19)$$

contains the $N - K$ orthonormal eigenvectors that correspond to the eigenvalue σ_n^2 .

It is easy to see that

$$\text{range}(\mathbf{A}) = \text{range}(\mathbf{V}_\alpha) . \quad (20)$$

The range space of \mathbf{V}_α is called the *signal subspace* and its orthogonal complement, the *noise subspace*, is spanned by \mathbf{V}_n . Define the $N \times N$ diagonal matrix

$$\mathbf{W}_0 = \mathbf{W} - \sigma_n^2 \mathbf{I}_N = \text{diag}(w_1 - \sigma_n^2, \dots, w_K - \sigma_n^2, 0, \dots, 0) . \quad (21)$$

From (12) and (13) we obtain

$$\mathbf{A} \mathbf{S} \mathbf{A}^T = \mathbf{V}_\alpha (\mathbf{W}_\alpha - \sigma_n^2 \mathbf{I}_K) \mathbf{V}_\alpha^T = \mathbf{V} \mathbf{A}_0 \mathbf{V}^T . \quad (22)$$

The linear multiuser GR for demodulating the k -th user's data bit in (7) is in the following form

$$\hat{b}_k = \text{sgn}(2\mathbf{Z}_k^T \mathbf{x} - \mathbf{x} \mathbf{x}^T + \mathbf{n}_{AF} \mathbf{n}_{AF}^T) \quad (23)$$

followed by a hard limiter, where $\mathbf{Z}_k \in L^N$. Next, we derive

expressions for the GR in terms of the signal subspace parameters (\mathbf{V}_α , \mathbf{W}_α , and σ_n^2).

B. Generalized Receiver (GR)

For better understanding (23), there is a need to recall the main statements of the generalized approach to signal processing in noise [14]–[18], based on which the GR is constructed.

There are two linear systems at the GR front end that can be presented as bandpass filters, namely, the preliminary filter (PF) with the impulse response $h_{PF}(\tau)$ and the additional filter (AF) with the impulse response $h_{AF}(\tau)$. For simplicity of analysis, we consider that these filters have the same amplitude-frequency responses and equal bandwidths. Moreover, a resonant frequency of the AF is detuned relative to a resonant frequency of PF on such a value that signal cannot pass through the AF.

Thus, the signal and noise can be appeared at the PF output and *the only noise* is appeared at the AF output. It is well known fact that if a value of detuning between the AF and PF resonant frequencies is more than $4 \div 5 \Delta f_a$, where Δf_a is the signal bandwidth, the processes forming at the AF and PF outputs can be considered as independent and uncorrelated processes (in practice, the coefficient of correlation is not more than 0.05), but, in the case of signal absence in the input process the statistical parameters at the AF and PF outputs will be the same, because the same noise is coming in at the AF and PF inputs and we may think that the AF and PF do not change the statistical parameters of input process, since they are the linear GR front end systems. By this reason, the AF can be considered as a generator of reference sample with *a priori* information a “no” signal is obtained in the additional reference noise forming at the AF output.

There is a need to make some comments regarding the noise forming at the PF and AF outputs. If the Gaussian noise given by (1) comes in at the AF and PF inputs (the GR linear system front end), the noise forming at the AF and PF outputs is Gaussian, too, because AF and PF are the linear systems and, in a general case, takes the following form:

$$n_{k_{PF}}(t) = \int_{-\infty}^{\infty} h_{PF}(\tau) n_k(t - \tau) d\tau \quad (24)$$

and

$$n_{k_{AF}}(t) = \int_{-\infty}^{\infty} h_{AF}(\tau) n_k(t - \tau) d\tau . \quad (25)$$

If, for example, AWGN with zero mean and two-sided power spectral density $0.5N_0$ is coming in at the AF and PF inputs (the GR linear system front end), then the noise forming at the AF and PF outputs is Gaussian with zero mean and variance given by [15, pp.264–269]

$$\sigma_n^2 = \frac{N_0 \omega_0^2}{8\Delta_F} , \quad (26)$$

where, in the case if the AF (or PF) is the RLC oscillatory circuit, then the AF (or PF) bandwidth Δ_F and resonance frequency ω_0 are defined in the following manner

$$\Delta_F = \pi\beta \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}} , \quad (27)$$

where

$$\beta = \frac{R}{2L} . \quad (28)$$

The correlation matrix of the signature waveforms is defined as

$$\mathbf{R} = \mathbf{A}^T \mathbf{A} . \quad (29)$$

Since

$$\text{rank}(\mathbf{A}) = K , \quad (30)$$

it follows that \mathbf{R} is invertible.

Henceforth, let the user 1 be the user of interest. The GR (23) is designed to eliminate the MAI caused by other users, at the expense of enhancing the ambient noise. Assume that the weight vector $\mathbf{Z}_1 = \mathbf{g}_1$ given by

$$\mathbf{g}_1 = \sum_{k=1}^K [\mathbf{R}^{-1}]_{1k} \mathbf{a}_k , \quad (31)$$

where $[\mathbf{R}^{-1}]_{1k}$ denotes the (i, j) -th element of the matrix \mathbf{R}^{-1} .

The weight vector in (31) is characterized by the following statements:

- 1) The vector \mathbf{g}_1 in (31) is the unique signal

$$\mathbf{g}_1 \in \text{range}(\mathbf{V}_\alpha) , \quad (32)$$

such that

$$\mathbf{g}_1^T \mathbf{a}_1 = 1, \quad \text{and} \quad \mathbf{g}_1^T \mathbf{a}_k = 0, \quad \text{for } k = 2, \dots, K . \quad (33)$$

It is obvious, since

$$\text{rank}(\mathbf{V}_\alpha) = K , \quad (34)$$

the vector \mathbf{g} that satisfies the above conditions exists and is unique. It is seen from (31) that

$$\mathbf{g}_1 \in \text{range}(\mathbf{A}) = \text{range}(\mathbf{V}_\alpha) . \quad (35)$$

Moreover

$$\begin{aligned} \mathbf{g}_1^T \mathbf{a}_k &= \sum_{i=1}^K [\mathbf{R}^{-1}]_{1i} \mathbf{a}_i^T \mathbf{a}_k = \sum_{i=1}^K [\mathbf{R}^{-1}]_{1i} [\mathbf{R}]_{ik} \\ &= [\mathbf{R}^{-1} \mathbf{R}]_{1k} = \begin{cases} 1, & k = 1 \\ 0, & k = 2, \dots, K. \end{cases} \end{aligned} \quad (36)$$

Therefore, $\mathbf{g}_1 = \mathbf{g}$.

2) The vector \mathbf{g}_1 in (31) is the unique signal given by (32) that minimizes

$$f(\mathbf{g}) = E\left\{\left[\mathbf{g}^T \left(\sum_{k=1}^K S_k b_k \mathbf{a}_k\right)\right]^2\right\} \quad (37)$$

subject to

$$\mathbf{g}^T \mathbf{a}_1 = 1 \quad (38)$$

It is evident, since

$$\begin{aligned} f(\mathbf{g}) &= \mathbf{g}^T E\left\{\left[\left(\sum_{k=1}^K S_k b_k \mathbf{a}_k\right)\left(\sum_{k=1}^K S_k b_k \mathbf{a}_k\right)^T\right]\right\} \mathbf{g} \\ &= \mathbf{g}^T \left(\sum_{k=1}^K S_k b_k \mathbf{a}_k\right) \mathbf{g} = S_1^2 (\mathbf{g}^T \mathbf{a}_1)^2 + \sum_{k=2}^K S_k^2 (\mathbf{g}^T \mathbf{a}_k)^2 \\ &= S_1^2 + \sum_{k=2}^K S_k^2 (\mathbf{g}^T \mathbf{a}_k)^2 \end{aligned} \quad (38)$$

it then follows that for

$$\mathbf{g} \in \text{range}(\mathbf{V}_\alpha) = \text{range}(\mathbf{A}) \quad (39)$$

$f(\mathbf{g})$ is minimized if and only if (33) is satisfied. According to (36) we obtain (38) again.

3) The vector \mathbf{g}_1 in (31) is given in terms of the signal subspace parameters in the following form:

$$\mathbf{g}_1 = \frac{\mathbf{V}_\alpha (\mathbf{W}_\alpha - 4\sigma_n^4 \mathbf{I}_K)^{-1} \mathbf{V}_\alpha^T \mathbf{a}_1}{\mathbf{a}_1^T \mathbf{V}_\alpha (\mathbf{W}_\alpha - 4\sigma_n^4 \mathbf{I}_K)^{-1} \mathbf{V}_\alpha^T \mathbf{a}_1} \quad (40)$$

The proof is not difficult.

The canonical form of the linear minimum mean-square-error (MMSE) multiuser GR of user 1 has the form of (23) with the weight vector $\mathbf{Z}_1 = \mathbf{m}_1$, where $\mathbf{m}_1 \in L^N$ minimizes the MSE defined as

$$\text{MSE}(\mathbf{m}_1) = E\{(S_1 b_1 - \mathbf{m}_1^T \mathbf{x})^2\} \quad (41)$$

subject to

$$\mathbf{m}_1^T \mathbf{a}_1 = 1 \quad (42)$$

For the linear MMSE GR \mathbf{m}_1 is given in terms of the signal subspace parameters by

$$\mathbf{m}_1 = \frac{\mathbf{V}_\alpha \mathbf{W}_\alpha^{-1} \mathbf{V}_\alpha^T \mathbf{a}_1}{\mathbf{a}_1^T \mathbf{V}_\alpha \mathbf{W}_\alpha^{-1} \mathbf{V}_\alpha^T \mathbf{a}_1} \quad (43)$$

Since (23) is invariant to positive scaling, the two linear multiuser GRs given by (40) and (43) can be interpreted as follows. First, the received signal \mathbf{x} is projected onto the signal subspace to get a K -vector

$$\mathbf{q} = \mathbf{V}_\alpha^T \mathbf{x} \quad (44)$$

which clearly is a sufficient statistic for demodulating the K user's data bits. The signature waveform \mathbf{a}_1 of the user of interest is also projected onto the signal subspace to obtain

$$\mathbf{r}_1 = \mathbf{V}_\alpha^T \mathbf{a}_1 \quad (45)$$

The projection of the linear multiuser GR in the signal subspace is then a signal $\mathbf{H}_1 \in L^K$ such that the data bit is demodulated as

$$\hat{b}_1 = \text{sgn}(\mathbf{H}_1^T \mathbf{q}) \quad (46)$$

According to (40) and (43), the projections of the multiuser GR and linear MMSE GR in the signal subspace are given, respectively, by

$$\mathbf{H}_1^g = \begin{pmatrix} \frac{1}{w_1 - 4\sigma_n^4} & & \\ & \ddots & \\ & & \frac{1}{w_K - 4\sigma_n^4} \end{pmatrix} \times \mathbf{r}_1 \quad (47)$$

$$\mathbf{H}_1^m = \begin{pmatrix} \frac{1}{w_1} & & \\ & \ddots & \\ & & \frac{1}{w_K} \end{pmatrix} \times \mathbf{r}_1 \quad (48)$$

Therefore, the projection of the linear multiuser GRs in the signal subspace are obtained by projecting the signature waveform of the user of interest onto the signal subspace, followed by scaling the k -th component of this projection by a factor of $1/(w_k - 4\sigma_n^4)$ or $1/w_k$. Note that as $\sigma_n^2 \rightarrow 0$, the two linear GRs become identical.

Since the autocorrelation matrix \mathbf{Q} , and therefore its eigen-components can be estimated from the received signal, from the preceding discussion, we see that both the first GR and the linear MMSE GR can be estimated from the received signal with *a priori* knowledge of only the signature waveform and timing of the user of interest, i.e., they both can be obtained blindly.

IV. SIMULATION EXAMPLE

It is seen from the previous section that the linear multiuser GRs are obtained as long as the signal subspace components are identified. Modern subspace tracking algorithms are recursive in nature and update the subspace in a sample-by-sample fashion. In this paper, we adopt the recently proposed projection approximation subspace-tracking algorithm [24] for blind adaptive multiuser detection application. The advantages of this algorithm include almost sure global convergence to

the signal eigenvectors and eigenvalues, low computational complexity, and the rank tracking capability.

We provide a simulation example to illustrate the performance of the subspace-based blind adaptive linear MMSE GR. This example compares the performance of the subspace-based blind adaptive linear MMSE GR with the performance of the minimum-output-energy (MOE) blind adaptive detector proposed in [12]. It assumes a synchronous CDMA system with processing gain $N = 31$ and six users ($K = 6$). The desired user is user 1. There are four 10-dB MAIs and one 20-dB MAI, i.e., $S_k^2/S_1^2 = 10$, for $k = 2, \dots, 5$ and $S_k^2/S_1^2 = 100$, for $k = 6$.

The performance measure is the output SINR, defined as

$$\text{SINR} = \frac{E^2 \{ \mathbf{m}^T \mathbf{x} - \mathbf{x} \mathbf{x}^T + \mathbf{n}_{AF} \mathbf{n}_{AF}^T \}}{\text{Var} \{ \mathbf{m}^T \mathbf{x} - \mathbf{x} \mathbf{x}^T + \mathbf{n}_{AF} \mathbf{n}_{AF}^T \}}, \quad (49)$$

where the expectation is with respect to the data bits of MAIs and the noise. In the simulation, the expectation operation is replaced by the time averaging operation. For the projection approximation subspace with deflation subspace tracking algorithm, we found that with a random initialization, the convergence is slow. Therefore, in the simulations, the initial estimations of the eigencomponents of the signal subspace are obtained by applying singular value decomposition to the first 50 data vectors. The projection approximation subspace with deflation subspace tracking algorithm is then employed for tracking the signal subspace. The time-averaged output SINR versus number of iterations is shown in Fig. 1.

As a comparison, the simulated performance of the recursive least squared (RLS) version of the MOE blind adaptive detector is also shown in Fig. 1. It has been shown in [13] that the steady-state SINR of this algorithm is given by

$$\text{SINR}^\infty = \frac{\text{SINR}^*}{1 + d + d \times \text{SINR}^*}, \quad (50)$$

where SINR^* is the optimal SINR value, and

$$d = \frac{1 - \beta}{2\beta} N \quad (51)$$

and $0 < \beta < 1$ is the forgetting factor. Hence, the performance of this algorithm is upper bounded by $1/d$ when

$$\frac{1}{d} \ll \text{SINR}^*, \quad (52)$$

as is seen in Fig. 1.

Although an analytical expression for the steady-state SINR of the subspace-based blind adaptive GR is very difficult to obtain, as the dynamics of the projection approximation subspace with deflation subspace tracking algorithm are complicated, it is seen from Fig. 1 that with the same forgetting factor β , the blind adaptive GR well outperforms the RLS MOE detector.

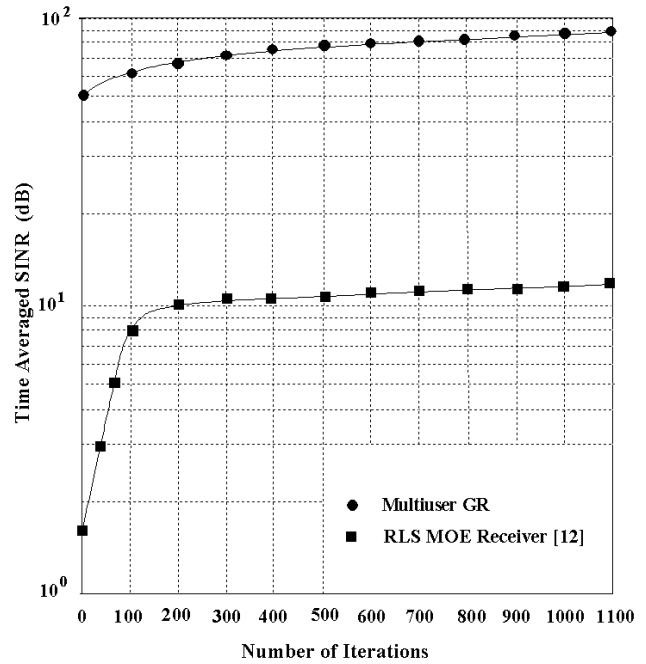


Fig. 1. Performance comparison between the space-based blind linear MMSE GR and RLS MOE detector

V. CONCLUSIONS

In this paper, we have developed the blind adaptive multiuser GR based on signal subspace estimation. Compared with the previous minimum-output-energy blind adaptive multiuser detection algorithm, it is seen that the proposed GR has better performance. We note from simulation example that the projection approximation subspace with deflation subspace tracking algorithm has a relatively slow convergence rate, which may pose a problem for a time-varying system. Nevertheless, subspace tracking is a very active research field in signal processing for wireless communications and it is anticipated that with the emergence of more powerful fast subspace trackers, for example [25], the performance of the subspace-based adaptive multiuser detectors will be improved.

ACKNOWLEDGMENT

This work was supported in part by the Kyungpook National University Research Grant, 2009.

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