Implementation of generalized detector for distributed sources using sensor arrays

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ABSTRACT

We consider a problem of detecting a random spatially distributed signal source by an array of sensors based on the generalized approach to signal processing in noise. We derive some generalized detector (GD) structures under several assumptions on the available statistics. The GD performance is evaluated and the effect of source angular spread is investigated. We notice the degrees of freedom of detection statistic distributions depend on both the signal angular spread and the number of data snapshots. At high signal-to-noise ratio and with small degrees of freedom, an increase of angular spread improves the detection performance. With large degrees of freedom the increase of angular spread reduces the detection performance. A comparison between GD and conventional beamformer is carried out by computer simulations. The results indicate a superiority of GD as the angular spread becomes large over the conventional beamformer detector.

Keywords: Generalized detector, distributed source, sensor array, generalized likelihood ratio test, adaptive coherence estimator detector, clutter, jamming.

1. INTRODUCTION

In the majority of cases, a large class of modern array processing techniques has been designed for point sources, i.e. spatially discrete sources of electromagnetic energy. In many applications, the transmitter is best modeled as a distributed, rather than a point source. The distributed sources appear to have certain angular spread with a mean direction of arrival (DOA). The point source model is only an approximation of the practical situation when there is a large distance between the source and the receiver array. The principal mechanism for making the source appear to be distributed in space is diffuse (irresolvable) and specular (resolvable) multipath caused by scattering of the propagation waves. For example, experimental results obtained in urban wireless communications reported significant angular scattering distributions due to local scattering and reflection from mobile stations\(^1\)\(^-\)\(^3\) and base stations\(^4\)\(^-\)\(^5\). The characterization of the power azimuth spectrum shows that angular spreads as large as 25° have been observed. The amount of angular spread is highly dependent on the scattering around the mobile, the height of the base station, and the distance between the base station and the mobile station. A secondary, but equally important, mechanism is transmitter motion. If the source moves significantly during the observation interval or coherent integration time, it will appear to be distributed rather that discrete. Angular spread has a significant impact on any array processing algorithms\(^6\), i.e. the signal-to-noise ratio (SNR) gain of the array reduces as the angular spread increases\(^7\), causing possible performance degradation. In passive array signal processing area, the problems under study concern the extraction of information from measurements using an array of sensors. Given the observations of the sensor outputs, the objective is to estimate the unknown parameters associated with the waveforms corrupted by noise. We start with a simple and computationally efficient detection scheme. If the “noise only” hypothesis is rejected, other algorithms are used to estimate the number of the sources and their unknown parameters, such as range and bearing. Prior work on distributed sources focuses primarily on source localization and DOA estimation\(^8\)\(^-\)\(^11\). Estimation of the number of distributed sources has also been studied in\(^8\). Subspace detectors have been studied in\(^12\)\(^-\)\(^13\) for the cases where the signal lies in a deterministic subspace. The case of detecting Gaussian signals with a low-rank covariance matrix and matched subspace detectors have been developed based on the generalized likelihood ratio test (GLRT). In both cases, the subspace in which the signals lie is assumed to be known. In other words, the dimension and rank of the signal subspace are assumed to be known a priori. This assumption does not hold in practical situations. The rank, orientation, and strength of the signal subspace vary along with the signal angular spread, DOA, and the energy distribution function. In this case, the unknown parameters may be estimated based on the maximum likelihood.

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principle. In this paper, we develop generalized detector (GD) constructed based on the generalized approach to signal processing in noise\textsuperscript{14-19} for distributed sources and study the GD performance depending on the angular spread. We first look at the case of known parameters. In this case, the GD can be approximated as a subspace beamformer. The detection performance depends on the detection statistic distribution and the sensor SNR. The degrees of freedom (DOF) of the detection statistics are determined by the angular spread and the number of data snapshots. If we fix the number of data snapshots, we show that the increase of angular spread reduces the mean of the detection statistics, which degrades performance, and at the same time reduces its variance, which improves performance. Within a certain range of DOF, the detection performance is improved with increasing the angular spread, up to a point, and then degrades slowly. We consider the case when the various parameters, such as signal direction, angular spread, power, and noise power are unknown. We derive the GLRT GD and evaluate its performance by computer simulation. We show the GLRT GD has a significant performance advantage compared with conventional beamformers as the angular spread becomes large.

2. PROBLEM STATEMENT

2.1 GD functioning principles

To better understand, we recall the main functioning principles of GD. The simple model of GD is represented in Fig.1. In this model, we use the following notations: MSG is the model signal generator (local oscillator), \(AF\) is the additional filter (linear system) and \(PF\) is the preliminary filter (linear system). A detailed discussion of the \(AF\) and \(PF\) can be found in\textsuperscript{15,17}. Consider briefly the main statements regarding the \(AF\) and \(PF\). There are two linear systems at the GD front end that can be presented, as bandpass filters: the \(PF\) with the impulse response \(h_{PF}(\tau)\) and \(AF\) with the impulse response \(h_{AF}(\tau)\). We think that these filters have the same amplitude-frequency responses and bandwidths. Moreover, a resonant frequency of the \(AF\) is detuned relative to \(PF\) one on such a value that is higher the signal bandwidth that signal cannot pass through the \(AF\). Thus, the signal and noise can be appeared at the \(PF\) output and the only noise is appeared at the \(AF\) output. It is well known, if a value of detuning between the \(AF\) and \(PF\) resonant frequencies is more than \(4 + 5\Delta \omega_a\), \(\Delta \omega_a\) is the signal bandwidth, the processes forming at the \(AF\) and \(PF\) outputs can be considered as independent and uncorrelated. In practice, the coefficient of correlation is not more than 0.05. In the case of a "no" signal in the input process, the statistical parameters at the \(AF\) and \(PF\) outputs will be the same, because the same noise is coming in at the \(AF\) and \(PF\) input, and we may think that the \(AF\) and \(PF\) do not change the statistical parameters of input process, since they are the GD front end linear systems. By this reason, the \(AF\) can be considered as a reference sample source with a priori information a "no" signal is obtained in the additional reference noise forming at the \(AF\) output. There is a need to make some comments regarding the noise forming at \(PF\) and \(AF\) outputs. If the Gaussian noise \(w(t)\) comes in at the \(AF\) and \(PF\) inputs (GD linear system front end), the noise forming at the \(AF\) and \(PF\) outputs is Gaussian, too, because \(AF\) and \(PF\) are the linear systems and, in a general case, take the following form:

\[
\begin{align*}
\mathcal{P}_{PF}(t) &= \int_{-\infty}^{\infty} h_{PF}(\tau)w(t-\tau)d\tau, \\
\mathcal{P}_{AF}(t) &= \int_{-\infty}^{\infty} h_{AF}(\tau)w(t-\tau)d\tau.
\end{align*}
\]  

(1)

If the additive white Gaussian noise (AWGN) with zero mean and two-sided power spectral density \(0.5N_0\) is coming in at the \(AF\) and \(PF\) inputs, then the noise forming at \(AF\) and \(PF\) outputs is Gaussian with zero mean and variance given by\textsuperscript{17} \(\sigma_n^2 = \frac{N_0\omega_0^2}{8\Delta \omega}\), where, in the case if \(AF\) (or \(PF\)) is the RLC oscillatory circuit, the \(AF\) (or \(PF\)) bandwidth \(\Delta F\) and resonance frequency \(\omega_0\) are defined in the following manner \(\Delta F = \pi\beta, \omega_0 = \frac{1}{\sqrt{LC}}\), \(\beta = \frac{R}{2L}\). The main GD functioning condition is an equality over the whole range of parameters between the model signal forming at the GD MSG output and the expected signal forming at the GD input liner system (the \(PF\) output). How we can satisfy this condition in practice is discussed in detail in\textsuperscript{15,17}. Detailed discussion about a choice of \(PF\) and \(AF\) and their amplitude-frequency responses is given in\textsuperscript{18-20} (see also http://www.sciedirect.com/science/journal/10512004, click "Volume 8, 1998", "Volume 8, Issue 3", and "A new approach to signal detection theory").

2.2 Signal model

Assume that we have an array with \(P\) sensors with an array response vector \(\mathbf{a}(\phi)\), where \(\phi\) denotes the azimuth. The array and all the sources are in the same plane. We assume a narrowband model for all signals and all the signals are defined
within the limits of baseband. The signal received by the array from a single source is modeled as

$$X_k = S_k + n_k, \quad k = 1, \ldots, N$$

(2)

where $N$ is the sample size and $X_k = [x_{k1}, \ldots, x_{kp}]$ is the array output at sample reading $k$; $S_k$ is the signal received at the array elements, assumed to be complex Gaussian with zero mean and covariance $E_S R_S$, where $E_S$ is the total signal energy and $R_S$ is defined later in (4); $n_k$ is the complex Gaussian noise with zero mean and covariance $R_n$. The signal and noise are uncorrelated from sample to sample. The correlation matrix of observation vector $X_k$ is given by

$$R_1 = E\{X_k, X_k^H\} = E_S R_S (\phi, \beta) + R_n,$$

(3)

where $(\cdots)^H$ denotes the complex conjugate transpose and the signal covariance matrix is given by

$$R_S (\phi, \beta) = \int_{-\pi}^{\pi} E(\theta; \phi, \beta) a(\theta) a^H (\theta) d\theta,$$

(4)

where $\phi$ is the source azimuth angle and $\beta$ is the source angular spread with $0 \leq \beta \leq 2\pi$; $a(\theta)$ is the array manifold at angle $\theta$; $R_S (\phi, \beta)$ is the normalized so that $\text{Tr}[R_S (\phi, \beta)] = P$; $\text{Tr} [\cdots]$ denotes the trace of a matrix, $E(\theta; \phi, \beta)$ is the spatial energy distribution of source at the azimuth $\phi$. More specifically, we may assume that $E(\theta; \phi, \beta) = E(\theta - \phi, \beta)$ with

$$\int_{-\pi}^{\pi} E(\theta; \phi, \beta) d\theta = 1.$$ The shape of the energy distribution function depends on the angular spread parameter $\beta$. If $\beta = 0^\circ$, $E(\theta; \phi, \beta)$ is a unit pulse. As $\beta$ increases, the energy distribution function becomes wider. For simplicity, we consider the uniformly distributed source model, i.e.

$$E(\theta; \phi, \beta) = \begin{cases} 1, \quad \theta \in [\phi - 0.5 \beta, \phi + 0.5 \beta] \\ \beta, \quad \theta \in [\phi - 0.5 \beta, \phi + 0.5 \beta] \\ 0, \quad \text{otherwise}. \end{cases}$$

(5)

One can also consider a Gaussian type distribution such that

$$E(\theta; \phi, \beta) = \frac{1}{\sqrt{2\pi \beta^2}} \exp \left( -\frac{(\theta - \phi)^2}{2\beta^2} \right), \quad \theta \in [\phi - 0.5 \beta, \phi + 0.5 \beta]$$

(6)

otherwise.

2.3 Signal subspace

By performing an eigen-decomposition of the matrix $R_S (\phi, \beta)$, we get

$$R_S (\phi, \beta) = V \Lambda V^H = (V_r \quad V_n) \begin{pmatrix} \Lambda_r & 0 \\ 0 & \Lambda_n \end{pmatrix} \begin{pmatrix} V_r^H \\ V_n^H \end{pmatrix},$$

(7)

where $V = (V_r \quad V_n)$, $\Lambda = \text{diag}(\Lambda_r, \Lambda_n)$ and $\Lambda_r = \text{diag}(\lambda_1, \ldots, \lambda_r)$. Here, $\Lambda_r$ consists of the $1 \leq r \leq P$ largest eigenvalues of $R_S (\phi, \beta)$ in descending order and $V_r = (v_1, \ldots, v_r)$ contains the corresponding orthonormal eigenvectors. If $R_S (\phi, \beta)$ is a low-rank matrix with rank $r$, then $R_S (\phi, \beta) = V_r \Lambda_r V_r^H$. More generally, we will assume that $R_S (\phi, \beta)$ can be approximated by a rank $r$ matrix. The number of dominant eigenvalues may be defined as the minimum number of eigenvalues whose sum exceeded $\pi P$, where $0 \leq r \leq 1$ is close to unity, for example, $r = 0.95$. Thus,

$$R_S (\phi, \beta) \approx V_r \Lambda_r V_r^H.$$

(8)

The range space of $V_r$ is called the signal subspace and its orthogonal complement, called the noise subspace, is spanned by $V_n$. The effective rank of subspace is the number of dominant eigenvalues. The case where $\beta = 0^\circ$ corresponds to a point source where the rank $r = 1$, and the signal covariance becomes $R_S (\phi, 0) = a(\phi) a^H (\phi)$.

2.4 Detection problem

The detection problem of a random Gaussian signal in the presence of noise and interference can be presented in the following form:
\[
X_k = \begin{cases} 
    n_k \rightarrow H_0, & k = 1, \ldots, N \\
    S_k + n_k \rightarrow H_1, & k = 1, \ldots, N.
\end{cases}
\] (9)

Covariance matrix of array output under the hypotheses \(H_0\) and \(H_1\) is \(R_0 = R_n\) and \(R_1\) given by (5), respectively. Let us assume, the interference has characteristics similar to that of the noise and can be absorbed into the noise vector \(n_k\). We assume that the GD input noise is spatially white or \(R_n = \sigma_n^2 I\), where \(\sigma_n^2\) is the noise variance and the noise power under the hypotheses \(H_0\) and \(H_1\) is not differed. If the noise is spatially colored or \(R_n = \sigma_n^2 R_m\), where \(R_m\) is a known positive definite Hermitian matrix, the detection is preceded by a prewhitening filter \(R_m^{-0.5}\). We have the binary hypotheses with \(X_k\) being a whitened data vector. We assume that the noise vectors \([n_1, \ldots, n_N]\) are independent and identically distributed (i.i.d.). By grouping all the \(N\) snapshots of observation vectors into an observation matrix \(X = [x_1, \ldots, x_N]\), we have \(X = S + N\), where the signal matrix \(S = [s_1, \ldots, s_N]\) and the noise matrix \(N = [n_1, \ldots, n_N]\). The detection statistics depend on a set of parameters

\[
p = \{E_S, \phi, \beta, \sigma_n^2\}. \tag{10}
\]

If all parameters are known, this is a standard detection problem whose optimal solution is the GD likelihood ratio\(^{14,16}\). If the parameters are unknown, we use the GLRT GD involving the replacement of unknown parameters by their maximum likelihood estimates under each hypothesis. Some GDs can be constructed depending on what parameters are known or unknown. We consider the GLRT GD in which the conventional beamformers are the counterparts with the same set of unknown parameters so that a fair comparison between the GLRT GD and conventional beamformer can be made. We consider the GD beamformer type 1 – all the signal parameters are known and this GD is used as a reference; the GLRT GD type 2 – the parameters \(\phi, \beta\) are unknown; the GLRT GD type 3 – the parameters \(E_S, \sigma_n^2, \phi, \beta\) are unknown; the GD beamformer type 4 – the parameter \(\phi\) is unknown; and the GD beamformer type 5 – the parameters \(\phi, E_S, \sigma_n^2\) are unknown. We carry out a comparative analysis between detectors of types 1, 2, 3, 4, and 5 with the corresponding conventional beamformer. The conventional beamformer is designed assuming the point sources (zero angular spread, i.e. \(\beta = 0^\circ\)).

### 3. GD DETECTORS

According to the generalized approach to signal processing in noise\(^{15,17}\), the detection problem for the GD can be presented in the following form:

\[
X_k = \begin{cases} 
    n_k \rightarrow H_0, & k = 1, \ldots, N \\
    S_k + \xi_k \rightarrow H_1, & k = 1, \ldots, N,
\end{cases}
\] (11)

where the elements of the vectors \(\xi_k\) and \(n_k\) are given by (1). Assume, the received signals under the hypotheses \(H_0\) and \(H_1\) are complex Gaussian with zero mean and covariance \(R_0\) and \(R_1\), respectively, and the probability density functions (pdfs) for the \(N\) observations are given by \(f(X|H_0) = [\pi^P | R_0 |]^{-N} \exp\{-N \times Tr[R_0 X Q]\}\) and \(f(X|H_1) = [\pi^P | R_1 |]^{-N} \times \exp\{-N \times Tr[R_1 X Q]\}\). Here,

\[
Q = \frac{1}{N} \sum_{k=1}^{N} [X_k^H][X_k^H]^H = \frac{1}{N} [X^H][X^H]^H \quad \text{and} \quad Q' = \frac{1}{N} \sum_{k=1}^{N} [X_k^H][X_k^H]^H = \frac{1}{N} [X^H][X^H] \tag{12}
\]

are the sample covariance matrices under the hypotheses \(H_1\) and \(H_0\), respectively. We emphasize that we do not impose the constraint that \(N \geq P\), i.e. detection can be carried out based on a single measurement (\(N = 1\)) or multiple measurements (\(N > 1\)). Based on the Neyman -Pearson theorem, when all the parameters are known, the GD test is the logarithm of the likelihood function ratio under the hypotheses \(H_1\) and \(H_0\), i.e.

\[
L(X) = Tr\left[2S^H[R_0^{-1} - R_1^{-1}]X^H - [X^H][R_0^{-1} - R_1^{-1}][X^H]^H + [R_0^{-1} - R_1^{-1}]Q' + N \ln \frac{R_0}{R_1}\right]. \tag{13}
\]

Next, we consider the case where some parameters are unknown. The unknown parameters under the hypotheses \(H_0\) and \(H_1\) are denoted by \(p_0\) and \(p_1\), respectively. In this case, the detection statistic is given by the GLRT\(^{20}\)
\[ l(X, \hat{p}_0, \hat{p}_1) = \frac{\max_{p_0} f(XH_1)}{\max_{p_0} f(XH_0)} \]  

or its logarithm

\[ L(X, \hat{p}_0, \hat{p}_1) = \text{Tr} \left\{ 2S^H \left[ R_0^{-1}(\hat{p}_0) - R_1^{-1}(\hat{p}_1) \right] X^{H1} - \left[ X^{H1} \right] [R_0^{-1}(\hat{p}_0) - R_1^{-1}(\hat{p}_1)] [X^{H1}]^H + [R_0^{-1}(\hat{p}_0) - R_1^{-1}(\hat{p}_1)] Q' + N \ln \frac{|R_0(\hat{p}_0)|}{|R_1(\hat{p}_1)|} \right\}, \]  

where \( \hat{p}_0 \) and \( \hat{p}_1 \) are the parameters maximizing the likelihood function under the hypotheses \( H_0 \) and \( H_1 \), respectively.

### 3.1 Subspace GD beamformer – known parameters

Ignoring the constant term \( N \ln \frac{|R_0|}{|R_1|} \) in (13), we can write

\[ L(X, p) = \text{Tr} \left\{ 2S^H W W^H X^{H1} - [X^{H1}] W W^H [X^{H1}]^H + W W^H Q' \right\}, \]  

where

\[ WW^H = R_0^{-1}(p) - R_1^{-1}(p) \]  

and \( p \) is defined in (10). The above decomposition of \( WW^H \) is possible because \( R_0^{-1} - R_1^{-1} \) is a non-negative definite matrix. In general, \( W \) is a \( P \times r \) matrix, where \( 1 \leq r \leq P \). The rank of \( W \) depends on rank of the signal covariance matrix \( R_S \).

Utilizing (5) and (8), we obtain \( R_1 \approx E_S V_r A_r V_r^H + R_0 \). Employing the matrix inversion lemma, we obtain \( R_1^{-1} \approx R_0^{-1} - R_0^{-1} V_r [E_S A_r + V_r^H R_0^{-1} V_r]^{-1} V_r^H R_0^{-1} \). Hence, we can rewrite (17) as follows:

\[ WW^H \approx R_0^{-1} V_r [E_S A_r + V_r^H R_0^{-1} V_r]^{-1} V_r^H R_0^{-1} \text{ and then, the subspace matrix is a } P \times r \text{ matrix} \]

\[ W = \frac{R_0^{-1} V_r}{\sqrt{E_S A_r + V_r^H R_0^{-1} V_r}} = [w_1, \ldots, w_r]. \]  

If the signal subspace is indeed low rank with rank \( r \), the approximation sign is replaced by an equality sign. This leads to an interpretation of the GD as a bank of beamformers. We emphasize, this subspace beamformer is an approximation to the true GD. In our case, the noise covariance matrix \( R_n = \sigma_n^2 I \) that leads us to a simpler form

\[ W \approx \frac{V_r}{4\sigma_n^4 \sqrt{\frac{V_r^H V_r}{4\sigma_n^4} + E_S A_r}}. \]  

We denote \( Z \) as an output matrix of beamformers \( Z = W^H X = [z_1, \ldots, z_N] \), where \( z_k = W^H x_k \). We will refer to this as a subspace beamformer to distinguish it from the conventional beamformer, where \( w \) is a \( P \times 1 \) vector. The GD output statistic is given by

\[ L(X, p) = \text{Tr} \left\{ 2Z S^H - YY^H + Q' \right\} = \sum_{k=1}^{N} \|2z_k s_k^H - z_k z_k^H + [x_k^H s_k^H] [x_k^H]^H \|^2. \]

### 3.2 GLRT GD – unknown parameter \( p = \{\phi, \beta, E_S, \sigma_n^2\} \)

#### 3.2.1 The parameter \( p = \{\phi, \beta\} \)

In this case, the unknown parameter \( p = \{\phi, \beta\} \) needs to be estimated. Note that the likelihood function of the GD background noise under the hypothesis \( H_0 \) is independent of the signal \( S \) and its parameters \( p \), and thus, it needs to be estimated only under the hypothesis \( H_1 \). The detection statistics are constructed as follows by maximizing argument \( p \):

\[ L(X, \hat{p}_1) = \text{Tr} \left\{ 2S^H [R_0^{-1} - R_1^{-1}(\hat{p}_1)] X^{H1} - [X^{H1}] [R_0^{-1} - R_1^{-1}(\hat{p}_1)] [X^{H1}]^H + [R_0^{-1} - R_1^{-1}(\hat{p}_1)] Q' + N \ln \frac{|R_0|}{|R_1(\hat{p}_1)|} \right\}, \]

where \( \hat{p}_1 = \arg \max_{\phi \in (-0.5\pi, 0.5\pi), \beta \in (0, \beta_0)} L(X, p_1) \). The estimation is done by a numerical maximization over the range \(-\pi \leq \phi \leq \pi \) or \(-0.5\pi \leq \phi \leq 0.5\pi \) (for a linear array), and \( 0 \leq \beta \leq \beta_0 \), \( \beta_0 \) is the predetermined angular spread range. The maximi-
zation is carried out by the following steps. Step 1: We divide a two-dimensional search range \( \{ \phi, \beta \} \) into small grids. The size of the grid \( \{ \delta \phi, \delta \beta \} \) is a fraction of an array beam width. We choose \( \delta \beta = \alpha_1 BW \) and \( \delta \phi = \alpha_2 BW \), where \( 0 < \alpha_1, \alpha_2 < 1 \). We find that \( \alpha_1, \alpha_2 = 0.5 \) produce quite reliable results. Step 2: For every pair \( \{ \phi, \beta \} \), \( R_S(\phi, \beta) \) is constructed based on (6). Step 3: The GD output statistic \( L [X(\phi, \beta)] \) is evaluated. Step 4: This process is repeated, and the largest value of \( L [X(\phi, \beta)] \) is selected.

### 3.2.2 The parameter \( p = \{ \phi, \beta, E_S, \sigma_n^2 \} \)

In this case, all the parameters are unknown. ML procedure requires a nonlinear optimization. The GD searches the global maximum of likelihood function over the unknown parametric sets. The maximization can be carried out as follows: 

\[
L(X, \hat{\theta}, \hat{\beta}, \hat{\sigma}_n^2, \hat{\sigma}_n^2, \hat{p}_1) = \max_{p_1} \max_{\rho \in \{0, \rho_0\}} L(X, E_S \hat{\sigma}_n^2, \hat{\sigma}_n^2, \hat{\sigma}_n^2, \hat{p}_1).
\]

We first maximize the likelihood function over \( E_S \sigma_n^2, \sigma_n^2 \), assuming that \( p_1 = \{ \phi, \beta \} \) is known. We maximize the result over \( \{ \phi, \beta \} \). The derivation of the maximum likelihood estimators of \( E_S \sigma_n^2, \sigma_n^2 \) is provided at the fixed \( p_1 \). The GLRT GD likelihood function takes the following form:

\[
l(X, \hat{\theta}, \hat{\beta}, \hat{\sigma}_n^2, \hat{\sigma}_n^2, \hat{p}_1) = \frac{1}{|E_S \hat{\sigma}_n^2 R_S(p_1) + I|^N} \left\{ \frac{\text{Tr}[Q] \text{Tr}[E_S \hat{\sigma}_n^2 R_S(p_1) + I]}{Q'} \right\}^{PN}.
\]

Taking the logarithm, we obtain the following GD output statistic

\[
L(X, \hat{\theta}, \hat{\beta}, \hat{\sigma}_n^2, \hat{\sigma}_n^2, \hat{p}_1) = PN \ln \left\{ \frac{\text{Tr}[Q] \text{Tr}[E_S \hat{\sigma}_n^2 R_S(p_1) + I]}{Q'} \right\} - N \ln |E_S \hat{\sigma}_n^2 R_S(p_1) + I|.
\]

\(
\hat{p}_1 = \arg \max_{-0.5 \pi < \phi < 0.5 \pi, 0 \leq \beta \leq \beta_0} L(X, \hat{\theta}, \hat{\beta}, \hat{\sigma}_n^2, \hat{\sigma}_n^2, \hat{p}_1).
\)

In summary, the maximization is carried out as follows. Step 1: Follow Steps 1 and 2 in 3.2.1. Step 2: For each pair of \( \{ \phi, \beta \} \), a one-dimensional search over \( \rho \) is carried out to select the maximal value of \( L(X, \{ \phi, \beta \}, E_S \sigma_n^2) \). The search can be implemented through a binary search or an exhaustive search and \( p_0 \) is a predetermined search range. Step 3: This process is repeated for all the values of \( \{ \phi, \beta \} \), and the largest value of \( L(X, \{ \phi, \beta \}, E_S \sigma_n^2) \) is selected.

### 3.3 GD Beamformer – unknown parameter \( p = \{ \phi, E_S, \sigma_n^2 \} \)

#### 3.3.1 The parameter \( p = \{ \phi \} \)

GD beamformer searches for the maximum energy by sweeping over all possible directions. This is the optimal solution (maximum likelihood) for point sources. If the source is distributed, only a fraction of the energy is captured by the beamformer. Degradation in performance will occur. For a linear array, the beamformer searches over \(-0.5 \pi \leq \phi \leq 0.5 \pi \). The GD output statistic takes the following form

\[
L(X) = ||2SWH (\hat{\phi})XH - XH [WH XH]H + WHQ' ||^2,
\]

where \( \hat{\phi} = \arg \max_{-0.5 \pi < \phi < 0.5 \pi} ||WH (\hat{\phi})X||^2 \) and \( a(\phi) \) is the steering vector pointing to direction \( \phi \). The implementation of this GD beamformer is quite straightforward. The GD beamformer \( w \) is a normalized steering vector. The maximal value of GD beamformer output statistic is chosen by sweeping the GD beamformer from \(-0.5 \pi \) to \( 0.5 \pi \).

#### 3.3.2 The parameter \( p = \{ \phi, E_S, \sigma_n^2 \} \)

GD beamformer searches the maximum energy output over the range from \(-0.5 \pi \) to \( 0.5 \pi \). The signal energy is divided by an estimate of the GD background noise variance that is calculated by projecting GD array output on the subspace orthogonal to the rank 1 steering vector. To exclude the signal power that “leaks” through the beams adjacent to the main beam, we estimate the GD background noise power from beams that are further away from the main beam. We can choose to exclude the \( \beta_0 / BW \) beams adjacent to the main beam, where \( \beta_0 \) is a predetermined angular spread range. This seemingly ad hoc approach is the modified maximum likelihood estimate of the GD background noise variance. It is well known that the GD background noise variance estimate is given by

\[
\sigma_B^2 = ||P_AQ'||^2 (P - r_A)^{-1},
\]

where \( P_A \) is the projection on the signal subspace \( A \) and \( r_A \) is the signal subspace rank. The accuracy of the GD background noise estimate will degrade when a smaller subspace is used due to a leakage of the signal power onto the noise sub-

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space. We construct a subspace \( \mathbf{B} \subseteq \mathbf{A} \) so that the signal power is excluded from the estimation i.e. the subspace \( \mathbf{B}(\phi, \beta_0) = \{ \mathbf{a}(\phi - 0.5\beta_0), \ldots, \mathbf{a}(\phi + 0.5\beta_0) \} \) is generated. The power that is projected onto the null space of \( \mathbf{B}(\phi, \beta_0) \) is considered to be the GD background noise power, i.e.

\[
\hat{\sigma}_g^2(\phi, \beta_0) = \text{Tr}\{\mathbf{P}_B(\phi, \beta_0)^\dagger \mathbf{Q}^\dagger \}(P - r_B)^{-1};
\]

(26)

\( \mathbf{P}_B(\phi, \beta_0) = \mathbf{I} - \mathbf{P}_B(\phi, \beta_0) \); \( r_B \) is the rank of \( \mathbf{B} \). The GD output statistic possessing CFAR properties is given by

\[
L(X) = \max_{\phi \in (-0.5\pi, 0.5\pi)} \frac{\text{Tr}\{2\bar{S}^{\dagger} \mathbf{P}_a(\phi) X^{\dagger} - X^{\dagger} \mathbf{P}_a(\phi) (X^{\dagger} X)^{\dagger} + \mathbf{P}_a(\phi) \mathbf{Q}^\dagger \}}{\hat{\sigma}_g^2(\phi, \beta_0)},
\]

(27)

where \( \mathbf{P}_a(\phi) \) is the projection matrix on \( \mathbf{a}(\phi) \). The implementation of this GD type involves three steps. **Step 1:** For every angle \( \phi \), we construct a projection matrix \( \mathbf{P}_a(\phi) \). The subspace \( \mathbf{B}(\phi, \beta_0) \) and null projection matrix \( \mathbf{P}_B(\phi, \beta_0) \) are constructed accordingly. **Step 2:** The GD output is evaluated based on (52) for every angle \( \phi \). **Step 3:** The largest value of \( L(X, \phi, \beta, \rho) \) is selected.

4. PERFORMANCE ANALYSIS: KNOWN PARAMETERS

4.1 SNR gain versus angular spread

SNR gain is defined as the ratio of the array output to the input SNR. For point sources, SNR gain depends on the number of sensors. For a linear array with \( P \) sensors SNR gain is equal to \( P \). We derive SNR gain for the subspace beamformers. The subspace GD beamformer output takes the following form \( z_k = 2\bar{S}^{\dagger} \mathbf{W}^{\dagger} \mathbf{W}^{\dagger} \mathbf{X}^{\dagger} - \mathbf{W}^{\dagger} \mathbf{W}^{\dagger} \mathbf{X}^{\dagger} + \eta_k \mathbf{W}^{\dagger} \eta_k^{\dagger} \). Let the output SNR can be presented as the ratio between the average signal and average background noise powers at the beamformer output \( \text{SNR}_{\text{out}}^{\text{GD}} = \text{Tr}\{\mathbf{W}^{\dagger} \mathbf{E}[\mathbf{s}^{\dagger} \mathbf{s}] \mathbf{W}\}/\text{Tr}\{\mathbf{W}^{\dagger} \mathbf{E}[\mathbf{Q}] \mathbf{W}\} = \text{Tr}\{\mathbf{E}_S \mathbf{W}^{\dagger} \mathbf{R}_S \mathbf{W}\}/\text{Tr}\{\mathbf{W}^{\dagger} \mathbf{R}_n \mathbf{R}_n^{\dagger} \mathbf{W}\} = \frac{\text{E}_S}{\sigma_n^2} \times \text{Tr}\{\mathbf{E}_S \mathbf{W}^{\dagger} \mathbf{R}_S \mathbf{W}\}/\text{Tr}\{\mathbf{W}^{\dagger} \mathbf{W}\} \). The input SNR is defined as \( \frac{\text{SNR}_{\text{in}}^{\text{GD}}}{\text{SNR}_{\text{out}}^{\text{GD}}} = \text{Tr}\{\mathbf{E}_S \mathbf{W}^{\dagger} \mathbf{R}_S \mathbf{W}\}/\text{Tr}\{\mathbf{W}^{\dagger} \mathbf{W}\} \). We are interested in how the SNR gain of the subspace GD beamformer varies with the signal angular spread. Note that \( \mathbf{W}^{\dagger} = 0.25\sigma_n^{-4} \mathbf{I} - [4\sigma_n^{-4} \mathbf{I} + \mathbf{E}_S \mathbf{R}_S(\beta)]^{-1} \mathbf{V}_r \times [4\sigma_n^{-4} \mathbf{I} + \mathbf{E}_S \mathbf{A}_r]^{-1} \mathbf{V}_r \). Then, the SNR gain is defined as \( \text{SNR}_{\text{out}}^{\text{GD}} / \text{SNR}_{\text{in}}^{\text{GD}} = \text{Tr}\{\mathbf{D} \mathbf{A}_r\}/\text{Tr}\{\mathbf{D}\} \). For simplicity, we assume that the dominant eigenvalues are approximately equal, i.e. \( \lambda_i = \frac{P}{r}, i = 1, \ldots, r \). This approximation is certainly not very accurate, but it reveals some insights of the SNR gain behavior. Therefore, we obtain \( \text{SNR}_{\text{out}}^{\text{GD}} / \text{SNR}_{\text{in}}^{\text{GD}} \approx P/r \). This result shows that the SNR gain for the subspace GD beamformer not only depends on the number of sensors \( P \) but on source angular spread \( \beta \) through the subspace rank \( r \) as well. It is a monotone decreasing function of \( r \). The following can be concluded as two special cases: 1) in the point source case \( \beta = 0^\circ \) or \( \mathbf{R}_S = \mathbf{A} \), we get the SNR gain equal to \( P \); 2) in the case \( \beta = 180^\circ \) or \( \mathbf{R}_S = \mathbf{I} \), we get the unit SNR gain.

4.2 Receiver operating characteristics (ROC)

We derive an analytical expression of pdfs under two hypotheses \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \). By \( \chi_2^2 (z) \) and \( f_\chi (z) \) we denote the central Chi-squared distribution with \( M \) degrees of freedom and the pdf of the central Chi-squared distribution with \( M \) degrees of freedom, respectively. Log-likelihood ratios at the hypotheses \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) can be represented by sum of quadratic forms in complex Gaussian random variables

\[
L(X, \mathcal{H}_0) = \mathbf{W}^{\dagger} \mathbf{Q}^{\dagger} - \mathbf{X}^{\dagger}_{\mathcal{H}_0} \mathbf{W}^{\dagger} \mathbf{X}_{\mathcal{H}_0} = \sum_{k=1}^N \eta_k \mathbf{R}_0^{-1} \eta_k - \sum_{k=1}^N \bar{s}_k \mathbf{R}_0^{-1} \bar{s}_k = \sum_{k=1}^N \left[ \sum_{j=1}^r \lambda_j \left| v_j^{\dagger} \eta_k \right|^2 - \sum_{j=1}^r \lambda_j \left| v_j^{\dagger} \bar{s}_k \right|^2 \right];
\]

(28)

\[
L(X, \mathcal{H}_1) = 2\bar{s}^{\dagger} \mathbf{R}_S \mathbf{X}_{\mathcal{H}_1} + \mathbf{W}^{\dagger} \mathbf{Q}^{\dagger} - \mathbf{X}^{\dagger}_{\mathcal{H}_1} \mathbf{W}^{\dagger} \mathbf{X}_{\mathcal{H}_1} = \sum_{k=1}^N \bar{s}_k \mathbf{R}_1^{-1} \mathbf{R}_0^{-1} \bar{s}_k + \sum_{k=1}^N \left[ \sum_{j=1}^r \lambda_j \left| v_j^{\dagger} \eta_k \right|^2 - \sum_{j=1}^r \lambda_j \left| v_j^{\dagger} \bar{s}_k \right|^2 \right],
\]

where \( \lambda_j \) are nonzero eigenvalues of the matrix \( \mathbf{W}^{\dagger} \mathbf{Q}^{\dagger} \), \( v_j \) are the associated eigenvectors. Thus, we can write \( L(X, \mathcal{H}_1) = \ldots \)
\[
\sum_{k=1}^{N} \sum_{j=1}^{r} \gamma_{jk} | z_{jk} |^2, i = 1, \ldots, r,
\]
where \( z_{jk} \) are independent zero mean and unit variance complex Gaussian random variables and \( \gamma_{jk}, j = 1, \ldots, r \) are the \( r \) real nonzero weights. More precisely,

\[
y_{j0} = E_x \lambda_j (E_x \lambda_j + 4\sigma_n^4)^{-1}, y_{j1} = 0.25 E_x \lambda_j \sigma_n^{-4}, j = 1, \ldots, r
\]
where \( \lambda_j \) are the eigenvalues of \( R_s \). As extension of the results in \( ^{22} \) from real to complex numbers, the quadratic form \( L(\mathbf{X}, H_i) \) is distributed approximately as the scaled Chi-squared random variable, or \( L(\mathbf{X}, H_i) \sim \delta_i \mathcal{X}_{M_i} \), where the scaling factor \( \delta_i \) and the degrees of freedom \( M_i \) are given by

\[
\delta_i = \sum_{j=1}^{r} \sum_{j=1}^{r} \frac{\gamma_{jj}^2}{\sum_{j=1}^{r} \gamma_{jj}^2}, \quad M_i = 2N \left[ \sum_{j=1}^{r} \gamma_{jj}^2 \right]^{-1} \left[ \sum_{j=1}^{r} \gamma_{jj}^2 \right]^{-1} \times \left[ \sum_{j=1}^{r} \gamma_{jj}^2 \right]^{-1}.
\]
Plugging (28) into the last expression, we obtain

\[
\delta_0 = \frac{\sum_{j=1}^{r} \left[ \frac{\text{SNR}^m \lambda_j}{\text{SNR}^m \lambda_j + 4\sigma_n^4} \right]^2}{\sum_{j=1}^{r} \left[ \frac{\text{SNR}^m \lambda_j}{\text{SNR}^m \lambda_j + 4\sigma_n^4} \right]^2}, \quad M_0 = 2N \left[ \frac{\sum_{j=1}^{r} \left[ \frac{\text{SNR}^m \lambda_j}{\text{SNR}^m \lambda_j + 4\sigma_n^4} \right]^2}{\sum_{j=1}^{r} \left[ \frac{\text{SNR}^m \lambda_j}{\text{SNR}^m \lambda_j + 4\sigma_n^4} \right]^2} \right] \frac{\sum_{j=1}^{r} \lambda_j^2}{\sum_{j=1}^{r} \lambda_j^2}.
\]

Utilizing the Jacobian transformation, we can show that the pdf of detection statistic \( L(\mathbf{X}, H_i) \) is \( | \delta_i |^{-1} f_{M_i}(\delta_i^{-1}) \). Note that the absolute sign can be ignored because \( \delta_i \) is a positive number. This leads to an approximation of the probability of false alarm \( P_{AF} \approx \int_{\mathcal{S}_g} f_{M_0} (z) dz \) and the probability of detection \( P_D \approx \int_{\delta_0 / \delta_1 \mathcal{K}_g} f_{M_i} (z) dz \), where the pdf is given by

\[
f_{M_i} (u) = 0.5^{0.5M_i} \left[ \Gamma(0.5M_i) \right]^{-1} u^{0.5M_i-1} \exp\{-0.5u\}.
\]

Let \( F_{M_i} (u) = \int_{0}^{u} f_{M_i} (z) dz \) be the probability distribution function of Chi-squared random variable with \( M \) degrees of freedom and \( F_{M_i}^{-1}(u) \) is the inverse probability distribution function. The probability of false alarm \( P_{AF} \) and the probability of detection \( P_D \) of quadratic form \( L(\mathbf{X}, H_i) \) can be written as

\[
P_{AF} \approx 1 - F_{M_i}(u_{\text{FA}}) \quad \text{and} \quad P_D \approx 1 - F_{M_i}(u_{\text{DET}} / \delta_i).
\]

For a fixed level of false alarm rate, we have a closed-form expression of the probability of detection

\[
P_D \approx 1 - F_{M_i}(\delta_0 F_{M_i}(1 - P_{FA}) / \delta_i).
\]

Computer simulation shows that the analytical result (the lines) is matched with the Monte Carlo simulation result (the markers) quite well for different number of measurements, signal angular spreads, and false alarm rates. The asymptotic performance of the probability of detection \( P_D \) as \( N \) is sufficiently large can also be obtained in that the Chi-squared pdf with \( M \) degrees of freedom tends to become a Gaussian normal pdf with mean \( M \) and variance \( 2M \). If \( M \) is sufficiently large \( F_{M_i}(u) \to \Psi[u - M(2M)^{-0.5}], \) where \( \Psi(u) = \int_{\infty}^{u} (2\pi)^{-0.5} \exp\{-0.5u^2\} du \). We obtain \( P_D \approx 1 - \Psi\left[ \frac{\delta_0 M_0}{\sqrt{2M_0 \Psi^{-1}(1 - P_{FA}) - M_1}} (2M_1)^{-0.5} \right] \).

### 4.3 Required SNR (RSNR)

The ROC describes a behavior of the probability of detection \( P_D \) as the SNR changes given the probability of false alarm \( P_{FA} \). To compare different GD performance it is sometimes convenient to define a scalar performance measure, rather than use the entire ROC curve. We define the RSNR as the SNR needed to produce the probability of target detection at a given false alarm rate. Another quantity of interest is the output SNR required to achieve the same target \( P_D \) for the fixed
From (69) we obtain \( \delta_i / \delta_0 = F_{M_i}^{-1}(1 - P_{FA}) / F_{M_0}^{-1}(1 - P_D) \). Simplification leads us to

\[
RSNR = \frac{P}{\sum_{j=1}^{r} \lambda_i^2 x \left[ \sum_{j=1}^{r} \frac{SNR_{M_j}}{\lambda_i + 4\sigma^2} \right]^2} \times \frac{F_{M_i}^{-1}(1 - P_{FA})}{F_{M_0}^{-1}(1 - P_D)}. \tag{33}
\]

The general behavior of the quantity \( RSNR \) depends on eigenvalues, which has certain distributions. Eq. (33) is quite complicated. To gain some insights of this quantity, we look at the case where all the principal eigenvalues of \( \mathbf{R}_s \) are approximately equal, i.e. \( \lambda_i = \lambda = P / r, i = 1, \ldots, r \). This approximation is certainly not accurate but serves for simplicity purposes to illustrate the performance of \( RSNR \) as the degrees of freedom changes. A more accurate result may be obtained by a numerical approach. With this assumption, we have \( M_0 \approx 2Nr, M_i \approx 2Nr \). Let \( v = 2Nr \) denotes the degrees of freedom, and after some transformations, we get \( RSNR = \left( v / 2P \right) \times \left[ F_{r}^{-1}(1 - P_{FA}) / F_{v}^{-1}(1 - P_D) - 1 \right] \). and the quantity \( RSNR \) is given as \( RSNR = SNR_{out} / SNR_{in} \approx N \times \left[ F_{r}^{-1}(1 - P_{FA}) / F_{v}^{-1}(1 - P_D) - 1 \right] \). The quantity \( RSNR \) that is a function of \( (v, P_D, P_{FA}) \) can be easily evaluated by numerical method. For a fixed \( P_{FA} \), Fig. 2 depicts the \( RSNR \) for different values of degrees of freedom \( v \) and \( P_{FA} \). To eliminate the effect of size of the linear array, we multiply the \( RSNR \) by array size \( P \). It is easy to see that the normalized \( RSNR \) (e.g., \( P \times RSNR \)) is a quantity that depends on the degrees of freedom \( v \), the probability of detection \( P_D \), and the probability of false alarm \( P_{FA} \). This observation indicates that although the increase of the angular spread causes a reduction of the \( SNR \) gain, it changes the distribution of the detection statistics by increasing its degrees of freedom. Figure 2 also explains the different behavior of detection performance presented in Figs. 3 and 4. GD operating in a meaningful detection range requires a large target \( P_D \). In this case, the \( RSNR \) performance improves as the degrees of freedom increase to a certain point, and then, it starts dropping. Large \( N \) or large angular spread gives rise to an asymptotic performance of the \( RSNR \). As the degrees of freedom become large, either due to a large number of measurements or large angular spread, the detection performance degrades, but the degradation is not significant.

5. NUMERICAL RESULTS

We present computer simulations to illustrate the analytical performance results discussed above. We consider a uniform linear array with \( P \) sensors and half wavelength spacing. The array manifold is given by \( \mathbf{a}(<\phi>) = [1, \exp\{j2\pi d\lambda^{-1} \sin <\phi>\}, \ldots, \exp\{j2\pi d\lambda^{-1} \sin <\phi>\}]^T \), where \( \lambda \) is the target return signal wavelength, \( d = 0.5\lambda \) is the element spacing, and \( <\phi> \) is the azimuth angle. We find that is convenient to normalize the angular spread by the array beamwidth. We choose \( P = 20 \). Thus, the beamwidth of this linear array is 6.03°. We assume that the source is located at 20° relative to the broadside of the array. Throughout the simulation, \( P_{FA} = 10^{-3} \) and \( P_D = 0.8 \). We are interested in the case if there are a few snapshots available for detection. In the simulation, we will study the cases where \( N = 1 \) and \( N = 20 \), respectively. More detailed discussion of detection by multiple measurements is in.

5.1 Parameters are known – ROC, RSNR, and SNR gain

Figure 3 depicts the probability of detection \( P_D \) for different values of angular spreads \( \beta \), the probability of false alarm \( P_{FA} \), and number of measurements \( N \). We see both the analytical results using the formulas presented earlier and the Monte Carlo trial results. We observe that there exists a very good match between two sets of results. We notice that the behavior of \( P_D \) curves is quite different when multiple measurements are used in comparison with that of the single measurement detection. For a single measurement, \( N = 1 \) there is a crossover point of ROC curves for different angular spreads. We observe that as the angular spread increases from 0° to 30°, the detection performance improves up to a point, and then, it starts decreasing. If multiple measurements are used, the detection performance drops as the angular spread increases. This phenomenon is due to the change of the pdf shape, whose degrees of freedom are affected by both the angular spread and the number of measurements. Alternative way to look this phenomenon is to study the RSNR versus the angular spread. Figure 4 shows the behavior of the \( SNR \) gain versus the angular spread. The \( SNR \) gain plot decreases mo-
notonically from 17 to 5 dB as the angular spread increases from $\beta = 0^\circ$ to $\beta = 360^\circ$. GD output $\text{RSNR}$ shows a performance improvement caused by the GD output statistics pdf change as the angular spread increases. As the angular spread and the effective rank of the GD subspace increases, the detection statistics pdf becomes a Gaussian distribution. Once the pdf is sufficiently close to Gaussian one, no more improvement occurs. The $\text{RSNR}$ plot, which is the difference of these two plots, is decreased up to a point and increased afterwards. Figure 3 demonstrates a superiority of GLRT GD employment in comparison with the conventional beamformer.

5.2 Detection with angular spread mismatch

We examine the effects of angular spread mismatch on GD performance. In Fig. 5, the upper plot depicts the $\text{SNR}$ gain loss factor defined as the ratio of the $\text{SNR}$ gain for a subspace GD beamformer using an incorrect angular spread to that of a beamformer that uses the correct angular spread of $20^\circ$. This is due to the fact that the GD background noise power projected on the detector subspace is proportional to the effective rank of the subspace that increases with the assumed angular spread. The lower plot depicts $\text{RSNR}$ loss factor as the assumed angular spread changes. GD with the correct angular spread has the best performance. The conventional beamformer detector assumed a zero angular spread and experienced a loss of approximately 7 dB in this case.

5.3 Detection performance versus angular spread

At the fixed $P_{fa} = 10^{-3}$ we study $P_D$. GLRT GD detector searches over ranges of direction $\phi$ and angular spread $\beta$ the maximal value of likelihood function. Searching is carried out over the range $-90^\circ \leq \phi \leq 90^\circ$ and $0 \leq \beta \leq \beta_0$, where $\beta_0$ is set to be $60^\circ$. Figures 6 and 7 depict $P_D$ and $\text{RSNR}$ versus angular spread for different detectors with single measurement, $N = 1$. The results show that when the angular spread is small there is a little difference between the GLRT GD and GD beamformer because the source is appeared to be a point-like source. The main beam of the GD beamformer is able to capture all the energy coming from the source. Actually, the GLRT GD has slightly worse performance than that of the GD beamformer due to the excessive search which results in an increase of detection threshold. As the angular spread gets larger, the performance of both GLRT GD and GD beamformer degrades. The degradation of GD beamformer is much more significant than that of GLRT GD. In the case of large angular spread the GLRT GD demonstrates the better $P_D$ in comparison with the GD beamformer. We plot the results in the case of known parameters to see how much the detection performance degrades than occurs for other suboptimal detectors. We study the GD performance if multiple measurements, $N = 20$ are used. We compare the $P_D$ and $\text{RSNR}$ when the parameters are unknown (see subsections 3.2 and 3.3) at $N = 20$ data snapshots. We see that the detection performance is appeared to be quite different than at $N = 1$. If $N = 20$, as the angular spread increases, the detection performance degrades monotonically. This observation suggests that large degrees of freedom caused either by multiple measurements or by a large angular spread, bring down the detection performance. The GD beamformer detection performance degrades faster in comparison with GLRT GD one. Simulation results in Figs. 8 and 9 also demonstrate the better GLRT GD and GD beamformer performance in comparison with conventional beamformers as the angular spread increases.

6. CONCLUSIONS

In this paper, we studied the GLRT GD and GD beamformer for spatially distributed signal sources. The analysis and computer simulation demonstrate that the GD performance is better in comparison with conventional beamformer one that are designed for point sources. The performance difference increases as angular spread increases.

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Figure 1. GD principal flowchart.

Figure 2. Normalized $RSNR$ versus degrees of freedom $v \in [1,50]$ at various $P_D$; $P_{FA} = 0.001$.

Figure 3. $P_D$ versus $SNR$; all parameters are known; the number of sensors $P = 20$.
Figure 4. SNR gain versus normalized angular spread for an array with $P = 20$ sensors.

Figure 5. $\text{RSNR}$ and $\text{SNR}$ gain loss factor versus assumed angular spread when there is an angular mismatch; the number of sensors $P = 20$; the signal angular spread $\beta = 20^\circ$.

Figure 6. $P_D$ versus normalized angular spread; the number of measurement is $N = 1, \text{SNR} = 2\text{dB}, P_{FA} = 0.001$.

Figure 7. $\text{RSNR}$ versus normalized angular spread; the number of measurement is $N = 1, P_D = 0.8, P_{FA} = 0.001$.

Figure 8. $P_D$ versus normalized angular spread; the number of measurements is $N = 20, \text{SNR} = -8\text{dB}, P_{FA} = 0.001$.

Figure 9. $\text{RSNR}$ versus normalized angular spread; the number of measurements is $N = 20, P_D = 0.8, P_{FA} = 0.001$. 