SIGNAL PROCESSING NOISE
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Additive and multiplicative noise in the information signal can significantly limit the potential of complex signal processing systems, especially when those systems use signals with complex phase structures. During the last few years, this problem has been the focus of much research, and its solution could lead to profound improvements in applications of complex signals and coherent signal processing.

*Signal Processing Noise* sets forth a generalized approach to processing signals in noise, an approach that represents a remarkable advance in signal processing and detection theory. It extends the boundaries of noise immunity, and systems constructed on this basis achieve better detection performance than systems currently in use. Featuring the results of the author’s own research, the book is filled with examples and applications, and each chapter contains an analysis of recent observations obtained by computer modelling and experiments.

Tables and illustrations clearly show the superiority of the generalized approach over both classical and modern approaches to signal processing noise. Addressing a fundamental problem in complex signal processing systems, this book offers not only theoretical development, but also practical recommendations for raising noise immunity in a wide range of applications.

**FEATURES**
- Describes a new, generalized approach to signal processing noise that represents a major advance in the field
- Shows that it may be possible to raise the upper bound of noise immunity in complex signal processing systems
- Details theories particularly useful in practice
- Includes observations and experimental results never before published
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