Non-blind beamforming and DOA estimation by generalized receiver in MIMO wireless communication systems

Jin Gui Liu and Vyacheslav Tuzlukov

Abstract—We investigate the generalized receiver (GR) constructed based on the generalized approach to signal processing in noise employing non-blind beamforming algorithms and direction of arrival (DOA) estimation, which is implemented by MIMO wireless communication systems. Three non-blind beamforming algorithms, namely, the least mean square (LMS), the recursive least square (RLS) and the sample matrix inverse (SMI) are compared under employment by GR. DOA estimation techniques are applied based on multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance technique (ESPRIT). We suppose several GR structures employing the above mentioned non-blind beamforming algorithms jointly with DOA estimation procedure. Comparative analysis of simulation results allows us to conclude that the performance curves of GR with considered non-blind beamforming algorithms are very close to each other. Also, simulation results demonstrate superiority in the output signal-to-interference-plus-noise ratio (SINR) under employment of GR with the discussed non-blind beamforming algorithms and DOA estimation in MIMO wireless communication systems in comparison with the Neyman-Pearson detector.

Keywords—Generalized receiver, non-blind beamforming, direction of arrival (DOA), multiple signal classification (MUSIC), signal-to-interference-plus-noise ratio (*SINR*), and root-meansquare error (RMSE).

I. INTRODUCTION

WITH the large demand for high data rate applications and improved signal quality over wireless channels many investigations have been carried out to satisfy the requirements of future wireless communication systems [1], [2]. One solution to these problems resides in the use of multiple antennas at the transmitter and/or receiver sides referred as the multiple-input multiple-output (MIMO) wireless communication systems. Many proposals within the framework of MIMO wireless communication systems have been introduced to improve the received signal quality and increase the high data rate over wireless links.

From the viewpoint of signal quality, it is known that the reception of multiple copies of the transmitted data based on

J. G. Liu is with the Jia Xing Zheng Xian Electrical Ltd., the joint Korean - Chinese company (e-mail: zao_q@hotmail.com).

employment of multiple antenna wireless communication systems improves the wireless communication system performance in comparison with the use of single antenna wireless communication systems. From viewpoint of channel capacity, it has been demonstrated that the use of multiple antennas has a great potential to increase substantially the data transmission rate converting wireless communication system channels from narrow to wide data pipes.

The problem of beamforming and direction of arrival (DOA) of the information signal is a very important problem in the MIMO wireless communication systems. In practice, the beamforming techniques are needed to eliminate effects of interference on MIMO wireless communication system performance. Adaptive beamforming is the mainstream technique for interference elimination by defining dynamically the optimal weight vectors of array antenna elements. Filtering procedures are not able to distinguish the desired signal in the noise and interference if they occupy the same frequency bandwidth.

The sources of desired and interfering signals are usually differed by spatial locations. The spatial separation can be exploited to distinguish a desired signal in the background of interfering signals using beamforming as a spatial filtering approach [3]. Beamforming is a versatile approach for wireless communications that is usually applied in antenna array systems for spatial filtering to separate signals having overlapping frequency content but originated from difference spatial locations. Adaptive beamforming is implemented in the case when the signal spatial locations are variable. Beamforming is employed using different algorithms with the purpose to change the weight vectors adaptively with respect to each antenna array element.

Adaptive beamforming algorithms can be categorized as the non-blind and blind algorithms depending on whether the reference signal is used or not. The non-blind beamforming algorithms update the weight vectors of antenna array to form a desired direction vector based on information about the information and reference signals. The least mean square (LMS), the recursive least square (RLS), and the sample matrix inverse (SMI) algorithms are categorized as the non-blind beamforming algorithms. The constant modulus algorithm (CMA), the spectral self-coherence restoral (SCORE), and the decision directed (DD) algorithms are examples of the blind beamforming algorithms.

Based on a priori knowledge about the information signal the non-blind beamforming algorithms can update the optimal weight vector with high accuracy. Therefore, the non-blind be-

This work was supported in part by the Kyungpook National University Research Fund, 2012-2014.

V. Tuzlukov is with the School of Electronics Engineering, College of IT Engineering, Kyungpook National University, Daegu, 702-701, South Korea (corresponding author, phone: +82-53-950-5509; fax: +82-53-950-5506; e-mail: tuzlukov@ee.knu.ac.kr)..

amforming technique attracts extensive research [4],[5]. When a priori knowledge about the direction of arrival (DOA) of the information signal is absent the non-blind beamforming technique is not valid unless the DOA estimation technique is applied. The main idea of DOA estimation technique is to exploit the spatial information in the data received by the antenna array. The beamforming based on the DOA estimation is a special approach to blind beamforming algorithms.

A great number of DOA estimation algorithms have been developed and categorized into two methods, namely, the conventional and subspace methods. The conventional method calculates a spatial spectrum and estimates DOAs by local maxima of the spectrum. Examples of this approach are the Bartlett and Capon methods [6]. However these methods suffer from lack of angular resolution. Therefore, the high angular resolution subspace methods such as the multiple signal classification (MUSIC) and the estimation of signal parameters via rotational invariance technique (ESPRIT) are widely used [7].

The generalized receiver (GR) has been constructed based on the generalized approach to signal processing (GASP) in noise and discussed in numerous journal and conference papers and some monographs, namely, in [8]-[26]. GR is considered as a combination of the correlation detector that is optimal in the Neyman-Pearson criterion sense under detection of signals with a priori known parameters and the energy detector that is optimal in the Neyman-Pearson criterion sense under detection of signals with a priori unknown parameters. The main functioning principle of GR is a matching between the model signal generated by the local oscillator in GR and the information signal by whole range of parameters. In this case, the noise component of the GR correlation detector caused by interaction between the model signal generated by the local oscillator in GR and the input noise and the random component of the GR energy detector caused by interaction between the energy of incoming information signal and the input noise are cancelled in the statistical sense. This GR feature allows us to obtain the better detection performance in comparison with other classical receivers.

In this paper, the GR employment with non-blind beamforming algorithms and DOA estimation technique in MIMO wireless communication systems is discussed. The simulation results demonstrate a performance superiority of wireless communication system constructed based on GR and applicability of the non-blind beamforming algorithms and DOA estimation techniques employed by GR for interference cancellation in comparison with wireless communication systems used the conventional detectors with the same non-blind-beamforming algorithms and DOA estimation procedures.

The remainder of this paper is organized as follows: the Section II presents the main GR functioning principles. The Section III delivers a description of non-blind beamforming algorithms employed by GR. Implementation of DOA estimation algorithm for wireless communications systems employed by GR is discussed in Section IV. The simulation results are presented in Section V. Some conclusions are discussed in Section VI.

II. GR FUNCTIONING PRINCIPLES

As we mentioned before the GR is constructed in accordan-

ce with GASP in noise [8]–[26]. The GASP introduces an additional noise source that does not carry any information about the signal with the purpose to improve a qualitative signal detection performance. This additional noise can be considered as the reference noise without any information about the signal to be detected.

The jointly sufficient statistics of the mean and variance of the likelihood function is obtained in the case of GASP employment, while the classical and modern signal processing theories can deliver only a sufficient statistics of the mean or variance of the likelihood function (no the jointly sufficient statistics of the mean and variance of the likelihood function). Thus, GASP implementation allows us to obtain more information about the input process or received signal. Owing to this fact, the receivers constructed based on GASP basis are able to improve the signal detection performance in comparison with other conventional receivers.

The GR consists of three channels (see Fig. 1): the correlation channel (the preliminary filter PF, multipliers 1 and 2, model signal generator MSG); the autocorrelation channel (PF, the additional filter AF, multipliers 3 and 4, summator 1); and the compensation channel (the summators 2 and 3 and accumulator 1).



Figure 1. Principal flowchart of GR

As we can see from Fig. 1, under the hypothesis \mathcal{H}_1 (a "yes" signal), the GD correlation channel generates the signal component $s_{\text{mod}_i}[k]s_i[k]$ caused by interaction between the model signal (the reference signal at the GR MSG output) and the incoming information signal and the noise component $s_{\text{mod}_i}[k] \times \xi_i[k]$ caused by interaction between the model signal $s_{\text{mod}_i}[k]$ and the noise $\xi_i[k]$ (the PF output). Under the hypothesis \mathcal{H}_1 , the GD autocorrelation channel generates the information signal energy $s_i^2[k]$ and the random component $s_i[k]\xi_i[k]$ caused by interaction between the information signal signal signal the noise $\xi_i[k]$. The main purpose of the GD compensation channel is to cancel the noise component $s_{\text{mod}_i}[k]\xi_i[k]$ of the GD correlation channel and the GD autocorrelation channel random component $s_i[k]\xi_i[k]$ based on the same nature of the noise $\xi_i[k]$.

For description of the GD flowchart we consider the discrete -time processes without loss of any generality. Evidently, this cancelation is possible only satisfying the condition of equality between the signal model $s_{\text{mod}_i}[k]$ and incoming signal $s_i[k]$ over the whole range of parameters. The condition $s_{\text{mod}_i}[k] = s_i[k]$ is the main functioning condition of the GD. To satisfy this condition, we are able to define the incoming signal parameters. Naturally, in practice, signal parameters are random. How we can satisfy the GD main functioning condition and define the signal parameters in practice if there is no a priori information about the signal and there is an uncertainty in signal parameters, i.e. information signal parameters are random, is discussed in detail in [8], [9].

Under the hypothesis \mathcal{H}_0 , a "no" information signal, satisfying the GD main functioning condition, i.e. $s_{\text{mod}_i}[k] = s_i[k]$, we obtain the background noise $\eta_i^2[k] - \xi_i^2[k]$ only at the GD output. Additionally, the practical implementation of the GD decision statistics requires an estimation of the noise variance σ_w^2 using the reference noise $\eta_i[k]$ at the AF output. AF is the reference noise source and the PF bandwidth is matched with the bandwidth of the information signal $s_i[k]$ to be detected. The threshold apparatus (THRA) device defines the GD threshold.

PF and AF can be considered as the linear systems, for example, as the bandpass filters, with the impulse responses $h_{PF}[m]$ and $h_{AF}[m]$, respectively. For simplicity of analysis, we assume that these filters have the same amplitude-frequency characteristics or impulse responses by shape. Moreover, the AF central frequency is detuned with respect to the PF central frequency on such a value that the information signal can not pass through the AF. Thus, the information signal and noise can be appeared at the PF output and the only noise is appeared at the AF output. If a value of detuning between the AF and PF central frequencies is more than $4 \div 5\Delta f_s$, where Δf_s is the signal bandwidth, the processes at the AF and PF outputs can be considered as the uncorrelated and independent processes and, in practice, under this condition, the coefficient of correlation between PF and AF output processes is not more than 0.05 that was confirmed by experiment in [27] and [28].

The processes at the AF and PF outputs present the input stochastic samples from two independent frequency-time regions. If the noise w[k] at the PF and AF inputs is Gaussian, the noise at PF and AF outputs is Gaussian, too, owing to the fact that PF and AF are the linear systems and we believe that these linear systems do not change the statistical parameters of the input process. Thus, the AF can be considered as a generator of reference noise with a priori knowledge a "no" signal (the reference noise sample). A detailed discussion of the AF and PF can be found in [9], [10].

The noise at the PF and AF outputs can be presented in the following form:

$$\begin{cases} w_{PF}[k] = \xi[k] = \sum_{m=-\infty}^{\infty} h_{PF}[m] w[k-m] ,\\ w_{AF}[k] = \eta[k] = \sum_{m=-\infty}^{\infty} h_{AF}[m] w[k-m] . \end{cases}$$
(1)

Under the hypothesis \mathcal{H}_1 , the signal at the PF output (see Fig.

1) can be defined as $x_i[k] = s_i[k] + \xi_i[k]$, where $\xi_i[k]$ is the observed noise at the PF output and $s_i[k] = h_i[k]s[k]$; $h_i[k]$ are the channel coefficients indicated here only in general case. Under the hypothesis \mathcal{H}_0 and for all *i* and *k*, the process $x_i[k] = \xi_i[k]$ at the PF output is subjected to the complex Gaussian distribution and can be considered as the independent and identically distributed (i.i.d.) process. The process at the AF output is the reference noise $\eta_i[k]$ with the same statistical parameters as the noise $\xi_i[k]$ (we make this assumption for simplicity).

The decision statistics at the GD output presented in [8], [9] is extended to the case of antenna array employment when an adoption of multiple antennas and antenna arrays is effective to mitigate the negative attenuation and fading effects. The GD decision statistics can be presented in the following form:

$$T_{GD}(\mathbf{X}) = \sum_{k=0}^{N-1} \sum_{i=1}^{M} 2x_i[k] s_{\text{mod}_i}[k] - \sum_{k=0}^{N-1} \sum_{i=1}^{M} x_i^2[k] + \sum_{k=0}^{N-1} \sum_{i=1}^{M} \eta_i^2[k] \gtrsim_{\mathcal{H}_0}^{\mathcal{H}} THR_{\text{GD}} , \qquad (2)$$

where $\mathbf{X} = [\mathbf{x}(0),...,\mathbf{x}(N-1)]$ is the vector of the random process forming at the PF output and THR_{GD} is the GD detection threshold. Under the hypotheses \mathcal{H}_1 and \mathcal{H}_0 , and when the amplitude of the signal is equal to the amplitude of the model signal, i.e. $s_{\text{mod}_i}[k] = s_i[k]$, the GD decision statistics $T_{GD}(\mathbf{X})$ takes the following form, respectively:

$$\begin{cases} \mathcal{H}_{1}: T_{GD}(\mathbf{X}) = \sum_{k=0}^{N-1} \sum_{i=1}^{M} s_{i}^{2}[k] + \sum_{k=0}^{N-1} \sum_{i=1}^{M} \eta_{i}^{2}[k] - \sum_{k=0}^{N-1} \sum_{i=1}^{M} \xi_{i}^{2}[k] ,\\ \mathcal{H}_{0}: T_{GD}(\mathbf{X}) = \sum_{k=0}^{N-1} \sum_{i=1}^{M} \eta_{i}^{2}[k] - \sum_{k=0}^{N-1} \sum_{i=1}^{M} \xi_{i}^{2}[k] . \end{cases}$$

$$(3)$$

In (3) the term $\sum_{k=0}^{N-1} \sum_{i=1}^{M} s_i^2[k] = E_s$ corresponds to the average signal energy, and the background noise is described by the term $\sum_{k=0}^{N-1} \sum_{i=1}^{M} \eta_i^2[k] - \sum_{k=0}^{N-1} \sum_{i=1}^{M} \xi_i^2[k]$ presents the background noise at the GD output. The GD background noise is a difference between the noise power forming at the PF and AF outputs. In the ideal case (equality of noise statistical parameters at the PF and AF outputs), this difference tends to approach zero in the statistical sense. Practical implementation of the GD decision statistics requires an estimation of the noise variance σ_w^2 using the reference noise at the AF output.

III. GR NON-BLIND BEAMFORMING

Let us consider general definitions. For an antenna array employed by adaptive beamforming algorithm the instant output vector $\mathbf{Z}[k]$ is defined as

$$\mathbf{Z}[k] = \mathbf{W}^{H}[k]\mathbf{X}[k] \quad , \tag{4}$$

where

$$\mathbf{X}[k] = \{X_1[k], X_2[k], \dots, X_M[k]\}, \ k = 0, \dots, N-1$$
(5)

r

is the discrete-time incoming signal vector received by antenna array or beamformer input signal; $\mathbf{W}^{H}[k]$ is the weight vector representing a series of discrete-time amplitude and phase coefficients that adjust accordingly the amplitude and phase of the information signal. The weight vector is updated by variety of adaptive beamforming algorithms.

The non-blind beamforming algorithms are the algorithms updating the weight vector based on the reference signal. The knowledge about the information signal is required to form a beam with the high gain towards the information signal and generate nulls towards the interfering signals at the same moment through the adjustment of the weight vectors. LMS, RLS and SMI algorithms are the typical non-blind beamforming algorithms employed by GR for interference cancellation. These three algorithms are based on the minimum mean square error (MMSE) criterion and have difference characteristics.

The main principle of MMSE is to minimize the mean square error between the beamformer output and the reference signal to update the weight vectors [29]

$$\mathbf{e}^{2}[k] = \left| \mathbf{d}[k] - \mathbf{W}^{H}[k]\mathbf{X}[k] \right|^{2}, \qquad (6)$$

where $\mathbf{e}[k]$ is the error vector between the beamformer output and reference signal or model signal vector (the MSG output); and $\mathbf{d}[k]$ is the reference signal vector in the beamformer. Taking the mathematical expectations of both sides in (6) and applying some basic algebraic transformations, we obtain

$$E\{\mathbf{e}^{2}[k]\} = E\{\mathbf{d}^{2}[k]\} - 2\mathbf{W}^{H}[k]\mathbf{r}[k] + \mathbf{W}^{H}[k]\mathbf{R}[k]\mathbf{W}[k]$$
(7)

where the covariance matrix $\mathbf{R}[k]$ and correlation matrix $\mathbf{r}[k]$ are given by

$$\mathbf{R}[k] = E\{\mathbf{X}[k]\mathbf{X}^{H}[k]\}, \qquad (8)$$

$$\mathbf{r}[k] = E\{\mathbf{X}[k]\mathbf{d}[k]\} .$$
(9)

The MMSE is given by setting the gradient vector of (7) with respect to the weight vector $\mathbf{W}[k]$, which is equal to zero

$$\nabla_{\mathbf{W}[k]} \left\{ E \left\{ \mathbf{e}^{2}[k] \right\} \right\} = -2\mathbf{r}[k] + 2\mathbf{R}[k]\mathbf{W}[k] = 0 \quad . \tag{10}$$

The solution of (10) can be presented in the following form

$$\mathbf{W}^{\mathrm{op}}[k] = \mathbf{R}^{-1}[k]\mathbf{r}[k] , \qquad (11)$$

that is well known as the optimum Wiener solution [29].

A. LMS Algorithm

<u>General Statements</u>. LMS algorithm is an iterative beamforming algorithm that uses the estimate of gradient vector from the available data based on the MMSE criterion and steepest descent method. LMS algorithm makes successive corrections of the weight vector in the negative direction of the gradient vector that should eventually lead to the optimal weight vector. By updating the weight vector that adjusts the phase and amplitude of the input signal, respectively, the output signal (beamformer output) will be close to the information signal by direction. Operation of LMS algorithm can be defined by the following form [4], [30]:

$$\mathbf{e}^{*}[k] = \mathbf{Z}_{GR}^{LMS}[k] - \mathbf{d}[k] , \qquad (12)$$

$$W[k+1] = W[k] + \mu e^{*}[k] Z_{GR}[k] , \qquad (13)$$

where μ is the step size adjusting the convergence rate of the LMS algorithm and * denotes the complex conjugate. The LMS algorithm is usually stable under the following condition $0 < \mu < \varphi$, where φ is the upper limit of the step size to ensure the algorithm stability.

As was shown in [6], the upper limit can be given by

$$\varphi = \frac{2}{\lambda_{\max}} \quad \text{or} \quad \varphi = 2\sum_{i=1}^{M} \lambda_i^{-1} = \frac{2}{tr\{\mathbf{R}[k]\}} \quad , \tag{14}$$

where λ_i is the eigenvalue of the received signal correlation matrix, and λ_{max} is the largest eigenvalue; M is the number of antenna array elements or the number of eigenvalues of the correlation matrix $\mathbf{R}[k]$; $tr\{\cdots\}$ means the trace of a matrix. The right side in (14) is a more conservation upper limit of the step size.

The eigenvalue spread of the matrix $\mathbf{R}[k]$ is inversely proportional to the convergence of the LMS algorithm. The widespread eigenvalues of $\mathbf{R}[k]$ may cause a slow convergence rate. Simultaneously, the variable signal environment leads to unstable eigenstructure of $\mathbf{R}[k]$ and causes a bad convergence performance for a fixed step size. Nowadays, the large number of variable step size LMS algorithms has been proposed in [30]–[32].

The generic approach is to control the step size by establish a function with respect to the mean squared error (MSE) at the beamformer output. The step size is increased when there is a large error, and the stable beamformer with a small error should decrease the step size. Another variable step size algorithm based on the signal-to-noise ratio (*SNR*) was proposed in [33]. This approach uses the antenna subarray elements to estimate a rough value of *SNR*. Based on defined *SNR*, a relational expression with the step size can be established.

$$\mu = \begin{cases} \mu_{\min} & \text{if } SNR > SNR_{\max} ; \\ f(SNR) & \text{if } SNR_{\max} > SNR > SNR_{\min} ; \\ \mu_{\max} & \text{if } SNR < SNR_{\min} ; \end{cases}$$
(15)

where μ_{max} and μ_{min} are the maximum and minimum step sizes; SNR_{max} and SNR_{min} are the limited maximal and minimal SNR defined at μ_{max} and μ_{min} , respectively, with the purpose to keep a stable convergence performance; f(SNR) is a function of SNR to control the step size adaptively. This function is a decreasing function since small step size is suitable for the large SNR. This variable step size algorithm based on SNR is available in the case when the information signal is changeable and possesses a good convergence performance.

GR with LMS Beamformer. In the case when there are inte-

rfering signals jointly with the received signal at the GR input, the LMS algorithm can be used at the GR output to cancel the interference. The structure of GR with LMS beamformer is shown in Fig.2. LMS beamformer cancels the interference component at the GR output using the model signal generated by GR MSG.



Figure 2. GR with LMS beamformer

Process forming at the GR output when the interference is taken into consideration is given by

$$Z_{\text{GR}} = \sum_{k=0}^{N-1} \sum_{i=1}^{M} \left\{ s_i[k] s_{\text{mod}_i}[k] - 2I_i[k] \xi_i[k] - I_i^2[k] + \eta_i^2[k] - \xi_i^2[k] \right\} ,$$
(16)

where $I_i[k]$ is the interfering signal, $I_i^2[k]$ is the interfering signal energy. The term $-2I_i[k]\xi_i[k]-I_i^2[k]$ in (16) indicates an interaction between the interference and noise that deteriorates the GR performance.

In GR with LMS beamformer, the reference signal of LMS beamforming algorithm $\mathbf{d}[k] = \mathbf{S}_{\text{mod}}^2[k]$ should be equivalent to the squared model signal at the GR MSG output due to the fact that the energy of the information signal is formed at the GR output. According to (4), (12), and (13), we can write

$$\mathbf{d}[k] = \mathbf{S}_{\text{mod}}^2[k] \; ; \tag{17}$$

$$\mathbf{Z}_{GR}^{LMS}[k] = \mathbf{W}^{H}[k]\mathbf{Z}_{GR}[k] ; \qquad (18)$$

$$\mathbf{e}^*[k] = \mathbf{W}^H[k] \mathbf{Z}_{\mathrm{GR}}[k] - \mathbf{S}_{\mathrm{mod}}^2[k] ; \qquad (19)$$

$$\mathbf{W}[k+1] = \mathbf{W}[k] + \mu \,\mathbf{e}^*[k] \mathbf{Z}_{GR}[k] \ . \tag{20}$$

Thus, the process forming at the GR output with LMS beamformer takes the form

$$\mathbf{Z}_{GR}^{LMS} = \sum_{k=0}^{N-1} \mathbf{W}^{H}[k] \mathbf{Z}_{GR}[k]$$
$$= \sum_{k=0}^{N-1} \{ \{ \mathbf{W}[k-1] + \mu \{ \mathbf{W}[k-1] \mathbf{Z}_{GR}^{H}[k-1] \} \}$$

$$-\mathbf{S}_{\text{mod}}^{2}[k-1] \{ \mathbf{Z}_{GR}[k-1] \}^{H} \mathbf{Z}_{GR}[k] \}.$$
(21)

The optimal weight vector obtained at the sample size k - 1 can be presented as

$$\mathbf{W}[k-1] = \mathbf{W}[k] = \mathbf{\mathcal{R}}^{-1}[k]\mathbf{\rho}[k] = \mathbf{\mathcal{R}}^{-1}[k-1]\mathbf{\rho}[k-1] , \qquad (22)$$

where $\Re[k] = \mathbf{Z}_{GR}[k]\mathbf{Z}_{GR}^{H}[k]$ is the autocorrelation matrix of the decision test statistics \mathbf{Z}_{GR} at the GR output, and $\rho[k] = \mathbf{Z}_{GR}[k]\mathbf{S}_{mod}^{2}[k] = \mathbf{Z}_{GR}[k]\mathbf{d}[k]$ is the correlation matrix between the decision statistics at the GR output and the LMS reference signal, respectively. Equation (22) is satisfied under the condition that the received signal is not varied from k - 1 to ksamples, i.e. $\mathbf{Z}_{GR}[k] = \mathbf{Z}_{GR}[k - 1]$.

Substituting (22) into (21), we obtain

$$\mathbf{Z}_{GR}^{LMS} = \sum_{k=0}^{N-1} \mathbf{W}^{H}[k] \mathbf{Z}_{GR}[k] = \sum_{k=0}^{N-1} \{ \{ \mathbf{\mathcal{R}}^{-1}[k] \mathbf{\rho}[k] \}^{H} \mathbf{Z}_{GR}[k] \}$$
$$= \sum_{k=0}^{N-1} \{ \{ \mathbf{Z}_{GR}[k] \mathbf{Z}_{GR}^{H}[k] \}^{-1} \mathbf{Z}_{GR}[k] \mathbf{S}_{mod}^{2}[k] \}^{H} \mathbf{Z}_{GR}[k]$$
$$= \sum_{k=0}^{N-1} \mathbf{S}_{mod}^{2}[k] = \sum_{k=0}^{N-1} \mathbf{S}^{2}[k] .$$
(23)

Therefore, we can see that the interference plus noise component in (16) is cancelled by the LMS beamformer and finally the GR output with LMS beamformer can be approximated by the decision test statistics defined in (3).

B. RLS Algorithm

<u>General Statements.</u> The convergence of the LMS algorithm depends on the eigenvalue of the correlation matrix $\Re[k]$. If the correlation matrix $\Re[k]$ has a wide range of eigenvalues, the algorithm converges slowly. The RLS algorithm is one of set of the algorithms that can improve the convergence performance by updating the weight vector based on MMSE criterion and Newton's method. The Newton's method is known as the gradient search method used to define the optimal weight vector by searching the performance surface which is defined with respect to an independent parameter [34]. This parameter is usually specified to be the MSE between the output and input in the adaptive signal processing area.

The common feature between the steepest descent method and the Newton's method is that both of them are the gradient searching methods and approach a direction of the optimal weight value by searching the performance surface. Newton's method updates the direction of searching in such a way that it points to the optimal weight vector forever, while the steepest descent indicates a negative direction of the gradient vector. Because of this, the RLS algorithm exhibits extremely fast convergence compared with LMS algorithm. However, this benefit leads us to increasing the computational cost.

RLS is realized by replacing the step gradient size μ in (23) by the gain matrix $\Re^{-1}[k]$ which is the inverse with respect to the matrix $\Re[k]$. Thus, the recursive equation for updating the weight vector can be defined as [29]

$$\mathbf{W}[k+1] = \mathbf{W}[k] + \mathbf{\mathcal{R}}^{-1}[k]\mathbf{e}^{*}[k]\mathbf{S}[k]$$
(24)

where $\boldsymbol{R}[k]$ is given by

$$\boldsymbol{\mathcal{R}}[k] = \gamma \boldsymbol{\mathcal{R}}[k-1] + \mathbf{S}[k]\mathbf{S}^{H}[k] = \sum_{j=0}^{k} \gamma^{k-j} \mathbf{S}[j]\mathbf{S}^{H}[j], \quad (25)$$

where γ is called the forgetting factor and intended to ensure that the data in the distant past are forgotten in order to allow the beamformer to follow the statistical variations of the observation data. The forgetting factor γ is a positive constant that should be chosen within the limits of the interval $0 < \gamma \le 1$.

We obtain the following recursive equation

$$\boldsymbol{\mathscr{R}}^{-1}[k] = \gamma^{-1} \{ \boldsymbol{\mathscr{R}}^{-1}[k-1] - \mathbf{q}[k] \mathbf{S}[k] \boldsymbol{\mathscr{R}}^{-1}[k-1] \}, \qquad (26)$$

using the Woodbury identity [29] with the purpose to derive an inverse of the covariance matrix $\Re[k]$, where $\mathbf{q}[k]$ is the gain vector that can be calculated as [35]

$$\mathbf{q}[k] = \frac{\gamma^{-1} \mathbf{\mathcal{R}}^{-1}[k-1] \mathbf{S}[k]}{1 + \gamma^{-1} \mathbf{S}^{H}[k] \mathbf{\mathcal{R}}^{-1}[k-1] \mathbf{S}[k]} = \mathbf{\mathcal{R}}^{-1}[k] \mathbf{S}[k] .$$
(27)

Thus, (24) can be rewritten as

$$\mathbf{W}[k+1] = \mathbf{W}[k] + \mathbf{q}[k]\mathbf{e}^*[k] . \qquad (28)$$



Figure 3. GR with RLS beamformer

<u>*GR* with *RLS* Beamformer</u>. The structure of GR employing RLS algorithm is presented in Fig. 3. Similar to the case of LMS algorithm, the RLS beamformer is applied at the GR output for interference cancellation. The beamformer input **S**[*k*] given by (24)–(27) can be considered as the GR output $\mathbf{Z}_{GR}[k]$. Simultaneously, the error in (28) is defined by (18) because the reference signal is the squared model signal $\mathbf{S}_{mod}^2[k]$ generated by the GR MSG. Thus, the process at the GR output under employment of the RLS beamformer can be presented in the following form:

$$\mathbf{Z}_{GR}^{RLS}[k] = \sum_{k=0}^{N-1} \mathbf{W}^{H}[k] \mathbf{Z}_{GR}[k]$$
$$= \sum_{k=0}^{N-1} \{ \mathbf{W}[k-1] + \mathbf{q}[k-1] \}$$
$$\times \{ \mathbf{W}[k-1] \mathbf{Z}_{GR}^{H}[k] - \mathbf{S}_{mod}^{2}[k-1] \} \}^{H} \mathbf{Z}_{GR}[k] , \qquad (29)$$

where the gain vector $\mathbf{q}[k]$ is defined by the GR output $\mathbf{Z}_{GR}[k]$ (see (27)) and the inverse matrix $\mathbf{\mathcal{R}}^{-1}[k]$ is obtained based on (26). Similar to (22) and (23), we can find that the interference plus noise component in (16) can be suppressed by the RLS beamformer and the final GR output with RLS beamformer is close to the information signal energy $\sum_{k=0}^{N-1} s^2[k]$.

C. SMI Algorithm

<u>General Statements.</u> SMI algorithm is another technique to improve the convergence performance when the eigenvalues of the correlation matrix $\Re[k]$ are widespread. The main principle of SMI algorithm is to inverse the sample matrix directly to obtain the optimal weight vector. In practice, the signals are unknown and the environment changes frequently. Thus, a block adaptive approach updating the weight vector to define new requirements imposed by varying conditions can be used to deliver a better performance in comparison with continuous approach [36].

The stability of the SMI algorithm depends on the ability to invert the correlation matrix $\Re[k]$ that may leads to computational complexities which are not easily overcome. However, SMI algorithm is a good choice in the case when high requirements of convergence performance are needed under the discontinuous varying environment. If a priori knowledge about information signal parameters is known the optimum weights can be determined directly based on the Wiener solution of (11). This approach is just the SMI algorithm or, namely, the direct matrix inverse (DMI).

When the input signal parameters are varied, the received data can be divided into several blocks that have stable spatial information and the optimal weights can be defined based on these blocks. This method is called the block adaptive method [36]. The block adaptive method updates the weight vector periodically by obtaining estimations of the covariance matrix $\Re[k]$ and correlation matrix $\rho[k]$ within the limits of the block size $L_2 - L_1$ considered as the observation interval. The estimations can be presented in the following form:

$$\begin{cases} \hat{\boldsymbol{\mathcal{R}}} = \sum_{l=L_1}^{L_2} \mathbf{Z}_{\text{GR}}[l] \mathbf{Z}_{\text{CR}}^H[l], \\ \hat{\boldsymbol{\rho}} = \sum_{l=L_1}^{L_2} \mathbf{d}[l] \mathbf{Z}_{\text{GR}}[l]. \end{cases}$$
(30)

Then, the weight vector can be estimated by the Wiener solution within the limits of the observation interval $N_2 - N_1$

$$\hat{\mathbf{W}}_{L_2-L_1}^{\text{opt}} = \hat{\boldsymbol{\mathcal{R}}}^{-1} \,\hat{\boldsymbol{\rho}} \quad . \tag{31}$$

SMI algorithm has the best convergence performance [29].However, it is not the best algorithm due to the limitations in computational complexity.

<u>GR with SMI Beamformer</u>. As we stated before, the SMI algorithm can also be applied under the GR employment. Figure 4 presents the GR flowchart with SMI beamformer. The beamformer input in (30) is the GR output $\mathbf{Z}_{GR}[l]$, and the estimation correlation matrix $\hat{\boldsymbol{\rho}}$ based on (17) and (30) can be written in the following form:



Figure 4. GR with SMI beamformer

Taking into consideration (16) and (31) we can write:

$$\mathbf{Z}_{\text{GR}}^{\text{SMI}}[k] = \sum_{l=L_1}^{L_2} \hat{\mathbf{W}}_{L_2-L_1}^H \mathbf{Z}_{\text{GR}}[l]$$
$$= \sum_{l=L_1}^{L_2} [\hat{\mathbf{x}}^{-1} \hat{\boldsymbol{\rho}}]^H \mathbf{Z}_{\text{GR}}[l] = \sum_{l=L_1}^{L_2} \mathbf{S}_{\text{mod}}^2[l] .$$
(33)

The error information is not used to update the weight vectors for this structure. This is the main difference between the SMI beamformer and the LMS and RLS adaptive beamformers. The observation interval $L_2 - L_1$ is used to provide a periodical determination of the weights if a nonstationary environment is anticipated.

IV. DOA ESTIMATION BY GR

DOA is an important parameter in the array signal processing. For smart antenna system, for example, in the wireless mobile communications, the DOA of signal is required for beamforming technique to form a beam in desired direction taking into account the multipath signal components.By this way, the signal-to-interference-plus-noise ratio (*SINR*) is improved by producing nulls towards the interfering signals.

In practice, DOA is usually unknown and should be estimated a priori based on the spatial information in the data received by antenna array. The performance of DOA estimation algorithm depends on many parameters such as the number of signals and their spatial distribution, the number of array elements and their spatial distance, the *SINR*, etc. The subspace algorithms, such as MUSIC and ESPRIT, are widely used and recognized as the effective estimation algorithms under different conditions.

The main idea of the subspace algorithms is to estimate the DOA by utilizing the characteristics of the noise and signal subspace derived from eigenvalue decomposition. Here, MUSIC and ESPRIT algorithms are employed by GR for DOA estimation. The GR flowchart with the DOA estimator is shown in Fig. 5. The DOA of signal is estimated at the PF output and is provided to the GR MSG with the purpose to generate the model signal.



Figure 5. GR with DOA estimator

A. MUSIC Algorithm

MUSIC algorithm is a high angular resolution technique for DOA estimation based on the eigenstructure of input covariance matrix and the decomposition of covariance matrix into two orthogonal matrices, i.e. the signal subspace and noise subspace. The main principle of MUSIC algorithm is to estimate the DOA by exploiting the orthogonal feature between the noise subspace and the signal subspace when the number of signals is less than the number of antenna array elements and the noise in each channel is uncorrelated and independent. The high resolution of MUSIC algorithm depends on the accuracy of array and requires a precise calibration of array.

The uniform linear array (ULA) is applied as an example to illustrate the algorithm procedure. If N is the number of signals, M is the number of antenna array elements, as shown in [7], the array covariance matrix can be determined in the following form

$$\mathbf{R}_{ZZ} = \mathbf{A}\mathbf{R}_{XX}\mathbf{A}^{H} + \sigma_{w}^{2}\mathbf{I} , \qquad (34)$$

where σ_w^2 is the noise variance, **I** is the $M \times M$ identity matrix; **A** is the $M \times N$ antenna array steering matrix or antenna array manifold

$$\mathbf{A} = \left[\mathbf{s}(\theta_1), \mathbf{s}(\theta_2), \dots, \mathbf{s}(\theta_N)\right]; \tag{35}$$

 \mathbf{R}_{XX} is the $N \times N$ received signal correlation matrix within the limit of the considered interval [0, N-1]

$$\mathbf{R}_{XX} = \mathbf{X}\mathbf{X}^H \quad , \tag{36}$$

where $\mathbf{X} = \{x_1[k], x_2[k], ..., x_N[k]\}^T$ is the received discrete-ti-

me signal matrix and $\theta_1, \theta_2, \dots, \theta_N$ denote the signal angles, *T* denotes the transpose matrix.

By the eigenvalue decomposition, the signal correlation matrix \mathbf{R}_{XX} has N eigenvalues and eigenvectors associated with signals and M - N eigenvalues and eigenvectors associated with the noise, respectively. Thus, the noise and signal eigenvector subspaces can be determined as

$$\mathbf{V}_W = [\mathbf{v}_1 \mathbf{v}_2 \cdots \mathbf{v}_{M-N}] \quad ; \tag{37}$$

$$\mathbf{V}_X = [\mathbf{v}_{M-N+1} \cdots \mathbf{v}_M] \quad . \tag{38}$$

Therefore, we can obtain the DOA, i.e. $\theta_1, \theta_2, ..., \theta_N$, by projecting the array steering vector $\mathbf{S}_{mod}(\theta)$ onto \mathbf{V}_W for all values of θ , and then the *N* values of θ can be found if the projection is zero [7]

$$\left\| \mathbf{s}^{H}(\theta_{n})\mathbf{V}_{W} \right\|^{2} = 0, \quad n = 0, 1, \dots N - 1$$
 (39)

where $\| \cdots \|^2$ is the square of two matrix norms.

Obviously, the steering vector $\mathbf{s}(\theta_n)$ of an arbitrary incident signal is orthogonal to the noise subspace. Thus, the MUSIC pseudo spectrum can be given by [7]

$$G_{\text{MUSIC}}(\theta) = \frac{1}{\left| \mathbf{s}^{H}(\theta) \mathbf{V}_{W} \mathbf{V}_{W}^{H} \mathbf{s}(\theta) \right|} , \qquad (40)$$

where $|\cdots|$ means the absolute value. By this way, the DOA of the signal can be estimated by searching the spectrum peak in the angular spectrum produced by (40). For GR with DOA estimator, the DOA of the desired signal is provided by MUSIC algorithm applied to GR MSG with the purpose to generate the model signal (see Fig. 5).

B. ESPIRIT Algorithm

The MUSIC algorithm involves an exhaustive search through all possible steering vectors to estimate the DOA with high computational complexity. ESPRIT is a subspace DOA estimation algorithm that can greatly reduce the computational cost and storage requirements [7].The main principle of ESPRIT algorithm is to divide the original antenna array into two subarrays with a translation invariance structure, and then a difference of the signal subspaces spanned by the data vectors associated with the subarrays will only be a rotational invariance factor containing the DOA information, which is exploited for the DOA estimation [37].

Consider ULA consisting of M elements. Two identical antenna subarrays have i elements that i < M for antenna subarrays that are not overlap 2i = M. The number of signals N is less than M. As was shown in [7], [37], the received data vectors of two antenna subarrays at the same instant of time are given by

$$\mathbf{Z}_1 = \mathbf{A}_1 \mathbf{X} + \mathbf{W}_1 , \qquad (41)$$

$$\mathbf{Z}_2 = \mathbf{A}_2 \mathbf{X} + \mathbf{W}_2 = \mathbf{A}_1 \mathbf{\Phi} \mathbf{X} + \mathbf{W}_2 , \qquad (42)$$

$$\mathbf{\Phi} = diag\{\exp[jvd\sin\theta_1]\cdots\exp[jvd\sin\theta_N]\},\quad(43)$$

where \mathbf{Z}_1 and \mathbf{Z}_2 are the $i \times 1$ received data vectors of antenna subarrays; \mathbf{A}_1 and \mathbf{A}_2 are $i \times N$ subarray manifold; \mathbf{W}_1 and \mathbf{W}_2 are the $i \times 1$ noise signal vectors of the subarrays; d is the distance between the antenna array elements; and v is the wave number. Based on (41)–(43) we can see that there is only a phase difference exp[$jvd\sin\theta_N$] between the received signals of two antenna subarrays, which is defined as rotational invariance. Thus, if we know the diagonal matrix $\mathbf{\Phi}$ given by (43) we can then estimate the target signal DOA.

Combining two data vectors (41) and (42) to form the complete received data vector of ULA, we obtain

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \mathbf{\Phi} \end{bmatrix} \mathbf{X} + \begin{bmatrix} \mathbf{W}_1 \\ \mathbf{W}_2 \end{bmatrix} = \mathbf{A}\mathbf{X} + \mathbf{W} .$$
(44)

The covariance matrix in (44) has N eigenvectors associated with signals and 2i - N eigenvectors associated with the noise. The subspace spanned by the column vectors of antenna array manifold and the signal subspace are orthogonal to the noise subspace, and the antenna array manifold is a full column rank matrix. By the uniqueness of orthogonal space, the subspace spanned by the column vectors of antenna array manifold is the same as the signal subspace. Thus, a unique $N \times N$ nonsingular matrix **T** satisfies the following equation

$$\mathbf{V}_{X} = \begin{bmatrix} \mathbf{V}_{X_{1}} \\ \mathbf{V}_{X_{2}} \end{bmatrix} = \mathbf{A}\mathbf{T} = \begin{bmatrix} \mathbf{A}_{1} \\ \mathbf{A}_{2}\mathbf{\Phi} \end{bmatrix} \mathbf{T} = \begin{bmatrix} \mathbf{A}_{1}\mathbf{T} \\ \mathbf{A}_{2}\mathbf{\Phi}\mathbf{T} \end{bmatrix}, \quad (45)$$

where \mathbf{V}_X is the signal subspace; \mathbf{V}_{X_1} and \mathbf{V}_{X_2} are the signal subspaces of the antenna sub arrays.

Then we can find that

$$\mathbf{V}_{X_2} = \mathbf{V}_{X_1} \mathbf{\Psi} \quad , \tag{46}$$

where Ψ is the rotational matrix given by

$$\Psi = \mathbf{T}^{-1} \mathbf{\Phi} \mathbf{T} \quad . \tag{47}$$

Equation (47) is considered as an eigenvalue decomposition of the rotational matrix Ψ . If we know the matrix Ψ , we can obtain the matrix Φ and enable to estimate the DOA of each signal.

The least square (LS) and the total least square (TLS) methods are commonly used to obtain the rotational matrix Ψ . Based on these two methods, there are two ESPRIT algorithms, i.e. LS-ESPRIT algorithm and TLS-ESPRIT algorithm. After obtaining the matrix Ψ the eigenvalues λ_{Ψ} can be defined and equal to the diagonal elements of the matrix such that

$$\begin{cases} \lambda_{\Psi_1} = \exp[jvd\sin\theta_1], \\ \vdots \\ \lambda_{\Psi_N} = \exp[jvd\sin\theta_N]. \end{cases}$$
(48)

Then we can estimate DOA using the following equation

$$\theta = \sin^{-1} \left[\frac{\arg(\lambda_{\Psi})}{vd} \right].$$
(49)

The ESPRIT DOA estimation algorithm does not require scanning over all possible directions. Because of this, the DOA of signal can be obtained without high computational efforts and large size of memory by the ESPRIT algorithm. However, there is a requirement about the array structure, for example, the antenna array should have at least two identical antenna subarrays.

V. SIMULATION AND DISCUSSION

Under simulation, we consider the antenna with ULA 8 elements and half signal wavelength distance. There are three incident signals. The first signal is the information signal arriving at 10° , the second and third signals are interfering signals arriving at -60° and 60° , respectively. These angles may be assumed to be unknown and estimated by DOA estimation algorithms. The interference-to-noise ratio (*INR*) is kept constant and equal to 5 dB. All signals are set as Gaussian random sequences with zero mean.

We compare GR and correlation receiver that is optimal in the Neyman-Pearson sense (NP receiver) by performance under identical input conditions. Figure 6 shows a comparison of



Figure 6. GR and NP receiver outputs; no interference

GR and NP receiver output signal modulus when there are no interfering signals and the input *SNR* is equal to 10 dB. The sample size *N* is equal to 10000. As follows from Fig. 6, the advantage of GR over NP receiver is evident. The output *SNR*s for GR and NP receiver are approximately equal to 3.52 dB and -2.4 dB.

Comparison of the output *SNR* versus the input *SNR* for GR and NP receiver is presented in Fig. 7. As was discuss in [8]–



Figure 7. The output *SNR* versus the input *SNR* for GR and NP receiver; no interference

[14], we can see when *SNR* is approximately less than $-1 \, dB$, the GR performance is low in comparison with the NP receiver one. Owing to selection of input conditions, this case does not correspond to receiver performance employed in practice because the probability of detection in this case is less than 0.1 and this is not practical case. In practice, the GR overcomes NP receiver by detection performance [15]–[26].

If the DOA of incident signals is unknown, the MUSIC and ESPRIT algorithm are employed for DOA estimation. Two interfering signals are considered and the DOA of all signals is estimated by two algorithms. The MUSIC angular spectrum and the estimated DOA of signals by ESPRIT algorithm are shown in Fig. 8 at the input SNR = 5 dB. Both algorithms have good resolution for DOA estimation.



Figure 8. The estimated DOA of signals: SNR = 5 dB



Figure 9. RMSE versus the input SNR

For details of performance analysis, the root-mean square error (RMSE) criterion [38]

$$RMSE = \sqrt{\sum_{k=0}^{N-1} \sum_{i=1}^{M} \frac{(\theta_i - \hat{\theta}_{k,i})^2}{NM}} = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{i=1}^{M} (\theta_i - \hat{\theta}_{k,i}) ,$$
(50)

can be employed to assess and compare the DOA estimation of different algorithms, where *N* is the sample size, *M* is the number of incident signals, θ_i is the *i*th DOA and $\hat{\theta}_{k,i}$ denotes the *i*th estimated DOA in the *k*th experiment. The RMSE versus the input *SNR* is demonstrated in Fig. 9. We see that the MUSIC algorithm has a higher resolution in comparison with ESPRIT algorithm at the low *SNR* values. At the high *SNR*, a difference between MUSIC and ESPRIT algorithms is negligible and both algorithms can be employed by GR in MIMO wireless communication system to generate the model signal in a general case.



Figure 10. GR array pattern with non-blind beamforming

Knowledge of signal DOA gives us an opportunity to employ the non-blind beamforming algorithms such as LMS, RLS, and SMI based on the reference signal generated by GR MSG. In the course of simulation, the forgetting factor of RLS is set equal to 0.99 and the block observation interval of SMI algorithm is [0, N-1]. The step size in LMS algorithm is chosen by (14) applying the variable step size approach based on *SNR*.

Figure 10 presents the antenna array pattern at the GR output with non-blind beamformer when the *SINR* is equal to 10 dB. At the target return signal direction equal to 10° , the beamformer can form a high gain. The nulls in the array pattern denote the direction of vectors of the interfering components owing to the GR processing.



Figure 11. The output SINR versus the input SINR

The output *SINR* as a function of the input *SINR* for GR and NP receiver with and without the LMS beamformer is presented in Fig.11. As we can see, a superiority of GR with nonblind beamforming algorithms in comparison with NP receiver is evident. Additionally, Fig. 11 presents the performance of interference cancellation by GR with non-blind beamforming algorithms in the term of the output *SINR* versus the input *SINR*. RLS and SMI algorithms have the better performance under interference cancelation in comparison with LMS algorithm.



Figure 12. GR output with interfering signals: a) without beamformer and b) with beamformer

Figure 12 shows the GR output without LMS beamformer, Fig. 12a, and with LMS beamformer, Fig. 12b, when the inter-

fering signals are present and the *SINR* is equal to 5 dB. The output *SNR* of GR with LMS beamformer is 13.98 dB while the output *SNR* of GR without LMS beamformer is 1.94 dB.



Figure 13. NP receiver output with interfering signals: a) without beamformer and b) with beamformer

Figure 13 presents the NP receiver output without LMS beamformer, Fig. 13a, and with LMS beamformer, Fig. 13b, when the interfering signals are present and the *SINR* is equal to 5dB The output *SNR* of NP receiver with LMS beamformer is equal to 3.98 dB and the output *SNR* of NP receiver without LMS beamformer is -11.2 dB. Under comparison of Figs.12 and 13, the advantage of GR with LMS beamformer over the NP receiver with LMS beamformer is evident.

VI. CONCLUSIONS

The GR with non-blind beamforming algorithms, namely, LMS, RLS, and SMI algorithms and DOA estimation procedures is investigated. LMS, RLS and SMI non-beamforming algorithms are employed by GR with the purpose to cancel interference. MUSIC and ESPRIT algorithms are subspace DOA estimation algorithms employed by GR to provide the GR MSG output with the required DOA information. Comparative analysis is carried out between the NP receiver and GR under the same initial conditions and demonstrates an applicability of the proposed non-blind beamforming and DOA estimation algorithms in GR that allows us to cancel interference. A great superiority of GR employment in MIMO wireless communication systems is evident under comparison between the GR and NP receiver in terms of the output *SNR*.

REFERENCES

- V. Tarokh, N. Seshadri, and A.R. Calderbank, "Space-time codes from high data rate wireless communication: Performance criterion and code construction," *IEEE Transactions on Information Theory*, 1998, Vol. IT-44, No. 2, pp. 744–765.
- [2] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs., Tech. Y.*, 1996, Vol. 1, No. 2, pp. 41–59.
- [3] B.D. Van Veen and K.M. Buckley, "Beamforming: A versatile approach to spatial filtering," *IEEE ASSP Magazine*, 1988, Vol. 5, No. 2, pp.4–24.

- [4] M. Usha, K. Naliji, and P. Ganesh, "Non-blind adaptive beamforming algorithms fro smart antennas," *International Journal of Research and Reviews in Applied Sciences*, 2011, Vol. 6, No. 4, pp. 491–496.
- [5] M. Yasin, A. Pervez, and D. Valiuddin, "Performance analysis of LMS and NLMS algorithms for a smart antenna systems," *International Journal of Computer Applications*, 2010, Vol. 4, No. 9, pp. 25–32.
- [6] H. L. Van Trees, Detection, Estimation and Modulation. Part IV: Optimum Array Processing, New York, John Willey & Sons Inc., 2002.
- [7] T.B. Lavate, "Performance analysis of MUSIC and ESPRIT DOA estimation algorithms for adaptive array smart antenna in mobile communications," in *Proceedings International Conference on Computer and Network Technology 2011*, April 15-18, 2011, Bangkok, Thailand, pp.153–158.
- [8] V. Tuzlukov, "A new approach to signal detection theory," *Digital Sig-nal Processing*, 1998, Vol. 8, No. 3, pp. 166–184.
- [9] V. Tuzlukov, Signal Detection Theory, New York, Springer-Verlag, 2001.
- [10] V. Tuzlukov, Signal Processing Noise, Boca Raton, CRC Press, 2002
- [11] V. Tuzlukov, *Signal and Image Processing in Navigational Systems*, Boca Raton, CRC Press, 2005.
- [12] V. Tuzlukov, Signal Processing in Radar Systems, Boca Raton, CRC Press, 2012.
- [13] V. Tuzlukov, "Generalized approach to signal processing in wireless communications: The main aspects and some examples," Chapter 11 in *Wireless Communications and Networks: Recent Advances*, Editor: Eksim Ali, Croatia, INTECH, 2012, pp. 305–338.
- [14] V. Tuzlukov, "Wireless communications:Generalized approach to signal processing," Chapter 6 in *Communication Systems: New Research*. Editor: Vyacheslav Tuzlukov, New York, NOVA Science Publisher, Inc., 2013, pp. 175–268.
- [15] V. Tuzlukov, "Optimal combining, partial cancellation, and channel estimation and correlation in DS-CDMA systems employing the generalized detector," WSEAS Transactions on Communications, 2009, Vol. 8, No.7, pp. 718–733.
- [16] V. Tuzlukov, "Multiuser generalized detector for uniformly quantized synchronous CDMA signals in AWGN channels," *Telecommunications Review*, 2010, Vol. 20, NO. 5, pp. 836–848.
- [17] V. Tuzlukov, "Signal processing by generalized detector in DS-CDMA wireless communication systems with frequency-selective channels," *Circuits, Systems and Signal Processing*, 2011, Vol. 30, No. 6, pp. 1197– 1230.
- [18] V. Tuzlukov, Signal processing by generalized receiver in DS-CDMA wireless communication systems with optimal combining and partial cancellation," *EURASIP Journal on Advances in Signal Processing*, 2011, Vol.2011,Article ID913 189, 15 pages, doi:10.1155/2011/913189; 2011.
- [19] V. Tuzlukov, "DS-CDMA downlink systems with fading channel employing the generalized receiver," *Digital Signal Processing*, 2011, Vol.21 No. 6, pp. 725–733.
- [20] V. Tuzlukov, "Design of optimal waveforms in MIMO radar systems based on the generalized approach to signal processing," WSEAS Transactions on Communications, 2012, Vol. 11, No. 12, pp. 448–462.
- [21] V. Tuzlukov, "Implementation of generalized detector in MIMO radar systems," WSEAS Transactions on Communications, 2013, Vol. 12, No. 3, pp. 107–120.
- [22] M. S. Shbat and V. Tuzlukov, "Spectrum sensing under correlated antenna array using the generalized detector in cognitive radio systems," *International Journal of Antennas and Propagation*, 2013, Vol. 2013, Article ID 853746, 8 pages, 2013. doi:10.1155/2013/853746.
- [23] V. Tuzlukov, "Bit error probability of quadriphase DS-CDMA wireless communication systems based on generalized approach to signal processing," *Telecommunications Review*, 2013, Vol. 23, No. 4, pp. 501–515.
- [24] M. S. Shbat and V. Tuzlukov, "Definition of adaptive detection threshold under employment of the generalized detector in radar sensor systems," *IET Signal Processing*, doi: 10.1049/iet-spr. 2013.0235, 2014 (in press).
- [25] V. Tuzlukov, "Error probability performance of quadriphase DS-CDMA wireless communication systems based on generalized approach to signal processing," WSEAS Transactions on Communications, 2014, Vol. 13, No.2, pp. 116–129.
- [26] M. S. Shbat and V. Tuzlukov, "Evaluation of detection performance under employment of the generalized detector in radar sensor systems," *Radioengineering*, 2014 (in press).
- [27] M. Maximov, "Joint correlation of fluctuative noise at the outputs of frequency filters," *Radio Engineering and Telecommunications*, 1956, No. 9, pp. 28–38.

- [28] Y. Chernyak, "Joint correlation of noise voltage at the outputs of amplifiers with non-overlapping responses," *Radio Physics and Electronics*, 1960, No. 4, pp. 551–561.
- [29] J. Litva and K. Y. Lo Titus, Digital Beamforming in Wireless Communications, Boston, London, Artech House, 1966.
- [30] R.W. Harris, D. M. Charbies, and F. A. Bishop, "A variable step (VS) adaptive filter algorithm," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 1986, Vol. ASSP-34, No. 2, pp. 309–316.
- [31] N. J. Bershad, "On the optimum gain parameter in LMS adaptation," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 1987, Vol. ASSP-35, No. 7, pp. 1065–1068.
- [32] R. Kong and E. W. Johnson, "A variable step size LMS algorithm," *IEEE Transactions on Signal Processing*, 1992, Vol-SP-40, No. 7, pp. 1633–1642.
- [33] S. Ikeda and A. Sugiyama, "An adaptive canceller with low signal distortion for speech codes," *IEEE Transaction on Signal Processing*, 1999, Vol. SP-47, No. 3, pp. 665–674.
- [34] J. C. Principe, N. R. Euliano, and W.C. Lefebvre, *Neural and Adaptive Systems: Fundamentals through Simulations*, New York, Willey & Sons, Inc., 1999.
- [35] R.L. Ali, A. Ali, A. Rehman, S.A. Khan, and S.A. Malik, "Adaptive beamforming algorithm for antijamming," *International Journal of Signal Processing, Image Processing and Pattern Recognition*, 2011, Vol. 4, No. 1, pp. 95–105.
- [36] K. K. Shetty, "A novel algorithm for uplink interference suppression using smart antennas in mobile communications," PhD dissertation, etd-04092004-143712, *The Florida State University*, 2004.
- [37] R.D. Chetan, A. N. Jadhav, and M. H. Swapnil, "Performance analysis of ESPRIT algorithm for smart antenna systems," *International Journal* of Communication Network and Security, 2011, Vol. 1, No. 3, pp. 34–37.
- [38] B. Liao, Z. Zhang, and S. Chan, "DOA estimation and tracking of ULAs with mutual coupling," *IEEE Transactions on Aerospace and Electronic Systems*, 2012, Vol. AES-48, No. 1, pp. 891–905.