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A New Approach to Signal Detection Theory

V. P. Tuzlukov

Institute of Engineering Cybernetics, National Academy of Sciences,
Surganov Street 6, Minsk 220012, Belarus
E-Mail: TUZLUK@newman.basnet.minsk.by

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This paper is devoted to fundamental problems of the generalized signal processing approach based on a seemingly abstract idea: to introduce an additional noise source which does not carry any information about a signal for the purpose of improving qualitative characteristics of information processing systems. Theoretical and experimental investigations carried out by the author lead to the conclusion that the suggested signal processing approach allows one to formulate a decision-making rule based on the definition of the jointly sufficient statistics of mean and variance of the likelihood function (or functional). Classical signal detection theory allows one to define only sufficient statistics of the likelihood function (or functional) mean. The presence of additional information about statistical characteristics of the likelihood function (or functional) leads to the best qualitative algorithms of signal detection in comparison with optimal signal detection algorithms of the classical theory. © 1998 Academic Press

1. INTRODUCTION

Analysis of the classical theory of signal detection in noise has demonstrated that the optimal signal detection algorithm (the correlation algorithm and matched filtering) has been synthesized under the following conditions [3–10,15,16]: a sample from noise space, where a "yes" signal may exist, is observed and analyzed. Optimal signal detection algorithm allows one to define only the sufficient statistic of likelihood function (or functional) mean using a mutual correlation function. However, it is well known that the most complete information about the likelihood function (or functional) can be derived only with jointly sufficient statistics of mean and variance [1,2]. But this is impossible under the above-mentioned initial conditions of the classical signal detection theory. The correlation signal detection algorithm and matched filtering are optimal and furnish the same result. The correlation algorithm and matched filtering are the best fits to classical signal detection theory under these conditions.

However, a modification of the initial prerequisites, arising from the fact that there is a noise space harboring a domain where a "yes" signal may exist and one where a "no" signal may exist that is known a priori, allows one to synthesize a signal detection algorithm generating jointly sufficient statistics of mean and variance of the likelihood function (or functional). It is reasonable to suggest that the more statistical parameters of the likelihood function (or functional) can be defined the more complete information about the likelihood function (or functional) can be derived. Furthermore, optimal signal detection algorithms are components of a synthesized signal detection algorithm. By virtue of this fact the last algorithm has been called the generalized signal detection algorithm. Investigations have indicated that the generalized signal detection algorithm is in excess of the optimal algorithm of the classical theory of signal detection by qualitative characteristics.

2. THEORY

2.1. Statement of the Problem

The simplest signal detection problem is the problem of binary detection in additive Gaussian noise with the zero mean and spectral power density $N_0/2$. An optimal detector may be realized as a matched filter or a correlation receiver. Detection quality depends on a standardized distance between two

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signal points of decision-making space. This distance is characterized by signal energies, the coefficient of correlation between signals, and the spectral power density of additive noise. Given that signal energies are the same, an optimal coefficient of correlation is equal to -1. Moreover, the signal shape is of no consequence. In spite of the fact that the classical signal detection theory is very orderly and smooth it cannot give the most complete answer to some of the following questions. Consider recent results [3–11, 13–16].

It is necessary to check a hypothesis H_0 that an input stochastic process is a normal process and has zero mean against an alternative H_1 that this process is normal but has a mean varied by a known law $a(t)$. In a statistical sense this problem is solved in the following manner. Uncorrelated coordinates

$$X_i = \sqrt{\lambda_i} \int_0^T X(t)F_i(t)dt$$

are considered as elements of an observed sample, where $X(t)$ is an input stochastic process within the limits of the time interval $[0, T]$ and λ_i and $F_i(t)$ are eigennumbers and eigenfunctions of the integral equation

$$F(t) = \lambda \int_0^T R(y - t)F(y)dy, \quad 0 < t < T,$$

where $R(t)$ is a known correlation function of additive noise. As a rule the first N coordinates are bounded.

When the hypothesis H_0 is considered the likelihood function of the observed sample $X_1, \ldots, X_N$ has the form (hereafter we suppose for simplicity that the variance of noise is equal to one)

$$f_{X|H_0}(X|H_0) = \frac{1}{(2\pi)^{N/2}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{N} X_i^2\right\}. \qquad (1)$$

Values determined in the form

$$X_i = \xi_i + a_i = \sqrt{\lambda_i} \int_0^T [\xi(t) + a(t)]F_i(t)dt$$

have been adopted as observed coordinates in considering the hypothesis H_1, where $\xi(t)$ is Gaussian noise. Then the likelihood function is

$$f_{X|H_1}(X|H_1) = \frac{1}{(2\pi)^{N/2}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{N} (X_i - a_i)^2\right\}. \qquad (2)$$

Using (1) and (2) the likelihood function ratio may be written

$$\frac{f_{X|H_1}(X|H_1)}{f_{X|H_0}(X|H_0)} = \frac{\exp\left\{-\frac{1}{2}\sum_{i=1}^{N} (X_i - a_i)^2\right\}}{\exp\left\{-\frac{1}{2}\sum_{i=1}^{N} X_i^2\right\}} = \exp\left\{\sum_{i=1}^{N} X_i a_i - \frac{1}{2}\sum_{i=1}^{N} a_i^2\right\} = l(X_1, \ldots, X_N) = C, \qquad (3)$$

where C is a constant which is determined by performance criteria of the decision-making rule. Taking the logarithm (3) we can write

$$\sum_{i=1}^{N} X_i a_i > K_N \Rightarrow H_1 \text{ or } \sum_{i=1}^{N} X_i a_i \le K_N \Rightarrow H_0, \; K_N = \ln C + \frac{1}{2}\sum_{i=1}^{N} a_i^2, \qquad (4)$$

where $\Sigma_{i=1}^{N} a_i^2 = E_a$ is a signal energy.

It is contended that the signal detection algorithm (4) is reduced to the calculation of $\Sigma_{i=1}^{N} X_i a_i$ and comparison with a threshold K_N. The signal detection algorithm has been set to be optimal for any of the selected performance criteria: Bayes' criterion, including as particular cases the posterior probability maximum and likelihood maximum; Neuman and Pierson's criterion; and the minimax criterion [11], and is called the correlation algorithm since a mutual correlation function is defined between input process $X(t)$ and signal $a(t)$.

During analysis of the signal detection algorithm (4), a property which is determined by noise immunity in conjunction with other factors is obtained. The essence of the analysis is reduced to substitution into (4) of the real values $X_i = a_i + \xi_i$ (hypothesis H_1) or $X_i = \xi_i$ (hypothesis H_0),

$$\sum_{i=1}^{N} X_i a_i = \sum_{i=1}^{N} a_i^2 + \sum_{i=1}^{N} a_i \xi_i \Rightarrow H_1,$$

$$\sum_{i=1}^{N} X_i a_i = \sum_{i=1}^{N} a_i \xi_i \Rightarrow H_0,$$

where the term $\Sigma_{i=1}^{N} a_i \xi_i$ is characterized as a noise component with zero mean and finite variance, determined

$$\text{as } N \Rightarrow \infty \quad \lim_{N\to\infty} \overline{\left\{\sum_{i=1}^{N} a_i \xi_i\right\}^2} = \frac{E_a N_0}{2}, \tag{5}$$

$N_0/2$ is a spectral power density of noise. The detection parameter

$$q = \sqrt{\frac{2E_a}{N_0}} \tag{6}$$

can be considered as qualitative characteristic of the signal detection algorithm (4) (hereafter it will be called signal-to-noise). Parameter (6) is very important and defines noise immunity together with other factors.

Consider the factors producing some questions under synthesis of the signal detection algorithm (4). It is known that $\Sigma_{i=1}^{N} X_i$ (sufficient statistic of the mean) and $\Sigma_{i=1}^{N} X_i^2$ (sufficient statistic of the variance) are jointly sufficient statistics characterizing a distribution law of the random values X_i. Sufficient statistics $\Sigma_{i=1}^{N} X_i^2$ of the likelihood functions $f_{X|H_1}(X|\ H_1)$ and $f_{X|H_0}(X|\ H_0)$ are reduced under synthesis of the signal detection algorithm (4). This is correct relative to the statistical theory of decision-making and writing form. But in a physical sense this causes a specific perplexity. The fact is that a "yes" signal (the mean a_i of the observed sample $X_1, \ldots, X_N$ is not zero) is meant in the numerator (3) and a "no" signal is meant in the denominator (3) under observation of the same coordinates. It would be hard to imagine another approach for the same sample $X_1, \ldots, X_N$ in the numerator and denominator of the likelihood function ratio. The question arises. Might a signal detection algorithm lossless of sufficient statistic of variance characterizing a distribution law be synthesized? Another factor producing some questions is that signal detection is performed with noise component $\Sigma_{i=1}^{N} a_i \xi_i$, which is caused by interaction between signal and noise. Variance of the noise component is proportional to signal energy, as follows from (5). The fact that signal-to-noise for the signal detection algorithm (4) defined by (6) is not proportional to $2E_a/N_0$ but is proportional to the square root of $2E_a/N_0$ is a consequence of this. Is this good or bad? One would think it is good given $2E_a/N_0 < 1$ but when the false alarm probability P_F is equal to 10^{-3}, for example, and given $q < 1$ the probability of correct detection P_D does not exceed 0.1, that is a practically inoperative region for a signal detection. Given $2E_a/N_0 > 1$ and $q = \sqrt{2E_a/N_0}$, P_D is smaller in comparison with the proportional function ($q = E_a/N_0$). This conclusion seems to be unusual but it is real, which will be shown in what follows.

Analyzing the signal detection algorithm (4) it can be noted that it is assumed to be optimal under the following conditions.

- The likelihood function (or functional) is taken using the same sample where the numerator assumes a "yes" signal and the denominator assumes a "no" signal. As this takes place, standard statistics are reduced and additional information is lost (sufficient statistic of likelihood function variance). The expression obtained maintains a calculation of sufficient statistic of the likelihood function mean only.
- In the theoretical aspect the signal detection algorithm (4) has not been synthesized rigorously because a mutual correlation function between input stochastic process X_i and signal a_i has been determined by the left part of (4); the left part (4) vanishes given "no" signal in input stochastic process X_i and any physical sense is wasted. In practice the signal detection algorithm (4) is implemented when a signal structure a_i is superseded by its model a_i^* in receiver ($a_i^* = ka_i$), where k is a coefficient of proportionality.
- Given that signal structure a_i is superseded by its model a_i^*, the noise component $\Sigma_{i=1}^{N} a_i^* \xi_i$ caused by interaction among signal model and noise is brought about.
- The variance of the marked noise component is proportional to the energy of the signal model, i.e., $E_a^* N_0/2$, where E_a^* is the energy of the signal model and $N_0/2$ is the spectral power density of noise.
- Signal detection algorithm (4) does not allow one to obtain a ratio between energy characteristics of signal and noise in the form $2E_a/N_0$. P_D is a function of square root of the ratio between signal and noise energy characteristics, i.e., signal-to-noise is equal to $\sqrt{2E_a/N_0}$.
- Signal detection algorithm (4) does not afford signal detection given that the signal structure is not in agreement with the structure of the signal model.
- In general a detector synthesized according to signal detection algorithm (4) must be a tracker but not a clear detector because an instant of signal appearance on the time axis is of unknown origin.

Considering the conditions of optimality for the signal detection algorithm (4) set forth briefly given that the same sample is observed in the numerator and denominator of the likelihood function, it is the author's opinion that it is necessary to make a critical review of initial prerequisites which constitute the foundation of classical signal detection theory.

2.2. Initial Prerequisites

Signal detection algorithm (4) has been synthesized based on a proposal that there is a frequency–time region Z of interferences where a signal may be present; i.e., there is an observed sample from this region relative to which it is necessary to make a decision "yes" signal (hypothesis H_1) or "no" signal (hypothesis H_0).

Modify initial prerequisites of the classical signal detection theory. Suppose there are two independent frequency–time interference regions Z and Z^* within a space A. Interferences from these regions adhere to the same distribution law with the same statistic parameters (the same distribution law and equality of statistic parameters have been selected for simplicity of analysis; in the general case distribution laws and statistic parameters may be unequal). A "yes" signal may be possible in the interference region Z as before. It is known a priori that a "no" signal is in the interference region Z^*. In what follows we will call the interference region Z^* and use the observed sample from this region as a reference. It is necessary to make a decision "yes" signal (hypothesis H_1) or "no" signal (hypothesis H_0) in an observed sample from the region Z by comparison of distribution law statistic parameters of this observed sample with parameters of the observed sample from the reference region Z^*.

The problem must be solved using the decision-making statistic theory. Thus, it is necessary to accumulate and compare statistic data defining statistic parameters of distribution laws of observed samples from two independent frequency–time regions Z and Z^*. If distribution law statistic parameters for two samples are the same or differ from each other with a given precision, then the decision "no" signal in the observed sample from the region Z (hypothesis H_0) is made. If distribution law statistic parameters of the sample from the region Z differ from parameters of the reference sample from the region Z^* by a value which exceeds the previously given precision, then the decision "yes" signal in the region Z (hypothesis H_1) is made.

2.3. Likelihood Ratio

Now, the problem is to obtain jointly sufficient statistics for a definition of statistic parameters of distribution laws. For this purpose let us refer to [17,22–28]. It is known [1,2] that sufficient statistics have been determined given that the likelihood function has an extremum. In the general case a condition of the likelihood function extremum by a parameter determined with a given precision is

$$\frac{\partial f_X(X_1, \ldots, X_N/\theta)}{\partial \theta} = 0,$$

where N is a sample size determining a given precision, and θ is the parameter to be determined. However, this equation has not been adopted practically. A simple mathematical procedure simplifies a representation of this equation. Since the logarithm is a monotonic function the extrema of the functions $f_X(X_1, \ldots, X_N)$ and $\ln f_X(X_1, \ldots, X_N)$ are reached at the same values of parameter θ. Therefore the likelihood function equation is usually written in the form

$$\frac{\partial \ln f_X(X_1, \ldots, X_N/\theta)}{\partial \theta} = 0. \tag{7}$$

As was shown in [2] using (2) and (7) it is easy to prove that $\Sigma_{i=1}^{N} X_i a_i$ and $\Sigma_{i=1}^{N} X_i^2$ are jointly sufficient statistics of likelihood function parameters (2) for the observed sample $X_1, \ldots, X_N$. Given (1) the likelihood function for the reference sample $\eta_1, \ldots, \eta_N$ is

$$f_\eta(\eta_1, \ldots, \eta_N) = \frac{1}{(2\pi)^{N/2}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{N} \eta_i^2\right\}, \tag{8}$$

where $\Sigma_{i=1}^{N} \eta_i^2$ is a sufficient statistic of likelihood function parameters for the reference sample $\eta_1, \ldots, \eta_N$.

In defining sufficient statistics using the samples $X_1, \ldots, X_N$ and $\eta_1, \ldots, \eta_N$, a problem of their comparison arises. Usually for this purpose a difference device is used (Fig. 1). Resulting sufficient statistics will be observed at the output of the difference device:

$$\ln f_X(X_1, \ldots, X_N) - \ln f_\eta(\eta_1, \ldots, \eta_N)$$
$$= \frac{1}{2}\left\{\sum_{i=1}^{N} 2X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 - \sum_{i=1}^{N} a_i^2\right\}. \tag{9}$$

It is acceptable to refer the last term of the right side

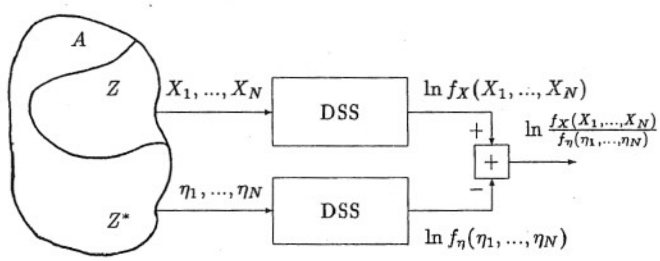

FIG. 1. Definition of jointly sufficient statistic.

of (9) to a threshold independent of the observed sample as in (4). Expression (9) obtained by definition of resulting sufficient statistic is a logarithm of the likelihood function. The signal detection algorithm based on two independent observed samples, one of which is the reference sample with a priori information "no" signal, follows from (9):

$$\ln f_X(X_1, \ldots, X_N)$$

$$- \ln f_\eta(\eta_1, \ldots, \eta_N) = \ln \left\{ \frac{f_X(X_1, \ \ldots, X_N)}{f_\eta(\eta_1, \ldots, \eta_N)} \right\}$$

$$= \frac{1}{2} \left\{ \sum_{i=1}^{N} 2\, X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 - \sum_{i=1}^{N} a_i^2 \right\} = \ln C$$

or

$$\sum_{i=1}^{N} 2X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 = K_N^*. \tag{10}$$

Proceeding from conventional concepts it follows that hypothesis H_1 ("yes" signal in the observed sample $X_1, \ldots, X_N$) is assumed if an inequality is performed,

$$\sum_{i=1}^{N} 2X_i a_i - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 > K_N^*, \tag{11}$$

and hypothesis H_0 ("no" signal in the observed sample $X_1, \ldots, X_N$) is assumed if an inverse inequality is performed. The first term of the left side of (11) is the signal detection algorithm (4). The more rigorous way of writing (11) based on the analysis performed in Section 2.1 is

$$\sum_{i=1}^{N} 2X_i a_i^* - \sum_{i=1}^{N} X_i^2 + \sum_{i=1}^{N} \eta_i^2 > K_N^*, \tag{12}$$

where a_i^* is a signal model. Analysis of the signal detection algorithm (12) performed on the same procedure as in Section 2.1 shows that considering the hypothesis $H_1(X_i = a_i + \xi_i)$ and given $a_i = a_i^*$ the left side of (12) has the form

$$2 \sum_{i=1}^{N} [a_i + \xi_i] a_i^* - \sum_{i=1}^{N} [a_i + \xi_i]^2 + \sum_{i=1}^{N} \eta_i^2$$

$$= \sum_{i=1}^{N} a_i^2 + \sum_{i=1}^{N} \eta_i^2 - \sum_{i=1}^{N} \xi_i^2,$$

where $\Sigma_{i=1}^{N} a_i^2 = E_a$ is signal energy, and $\Sigma_{i=1}^{N} \eta_i^2 - \Sigma_{i=1}^{N} \xi_i^2$ is background noise. Considering hypothesis $H_0(X_i = \xi_i,\, a_i = 0)$ and given $a_i^* = a_i$ the left side of (12) has the form $\Sigma_{i=1}^{N} \eta_i^2 - \Sigma_{i=1}^{N} \xi_i^2$.

Subsequent analysis of the signal detection algorithm (12) will be performed given $a_i = a_i^*$ solely. How this is done is discussed in Section 4. It is necessary to note that $\Sigma_{i=1}^{N} \eta_i^2 - \Sigma_{i=1}^{N} \xi_i^2 \Rightarrow 0$ as $N \Rightarrow \infty$ in the statistic sense since the processes ξ_i and η_i are uncorrelated and have the same spectral power density $N_0/2$ according to initial conditions.

By this means it was shown that algorithms for signal detection by both observed sample $X_1, \ldots, X_N$ and two independent observed samples $X_1, \ldots, X_N$ and $\eta_1 \ldots, \eta_N$ have the same approach and are determined by the likelihood function using the statistical theory of decision-making. The difference is that the numerator and the denominator of the likelihood function used for synthesis of the signal detection algorithm (4) involve the same observed sample (see (3)) but it is assumed that there is a "yes" signal in the numerator and "no" signal in the denominator. The numerator of the likelihood function used for a synthesis of the signal detection algorithm (12) involves the observed sample where a "yes" signal may be, and the denominator involves the reference sample relative to which it is known a priori that there is a "no" signal. On this basis it can be noted that sufficient statistic $\Sigma_{i=1}^{N} X_i a_i$ has been applied only for definition of the mean in the signal detection algorithm (4). In the signal detection algorithm (12) jointly sufficient statistics $\Sigma_{i=1}^{N} 2X_i a_i$ and $\Sigma_{i=1}^{N} (\eta_i^2 - X_i^2)$ are used for definition of the mean and variance of the likelihood function. This fact permits more complete information in decision-making in comparison with the algorithm (4). Heuristic synthesis of the algorithm (12) was first made in [20,21]. Signal detection algorithm (12) is free from a sequence of conditions unique to the signal detection algorithm (4). As algorithm (4) is a component of the algorithm (12) the latter has been called the generalized signal detection algorithm [17,20–42].

2.4. Physicotechnical Interpretation

Technical realization of independent samples from the regions Z and Z^* adhering to the same distribution law with the same statistic parameters is not difficult. Solutions of the detection problem for the signal $a(t)$ with additive Gaussian noise $n(t)$ are presented in [2–16]. The observed process $X(t)$ has been examined at the output of a receiver linear section which has an ideal amplitude–frequency response and bandwidth ΔF. It is assumed that noise at the input of the receiver linear section is white, having the correlation function $(N_0/2)\delta(t_2 - t_1)$, where $\delta(x)$ is a delta function. Signal $a(t)$ is assumed completely known and signal energy is equal to one. Spectral power density $N_0/2$ is considered as an a priori indeterminate parameter. The gain of the receiver linear section is equal to one. By analysis the problem has been reduced to a test of the complex hypothesis with solving function

$$\mathrm{Re}\int_0^T \dot{X}(t)\dot{a}^*(t)dt > C(\alpha)\sqrt{\int_0^T |\dot{X}(t)|^2 dt},$$

where $\dot{a}^*(t)$ is a filter matched with a signal, $C(\alpha)$ is a constant determined by P_F, and $\int_0^T |\dot{X}(t)|^2 dt$ is a statistic for the definition of a decision function. The signal detector synthesized in accordance with the above-mentioned decision function makes the P_F stable given that a power of noise is unknown and has the greatest P_D for any signal-to-noise.

Interpret this problem. Let us use two receiver linear sections for a statistic set. These receiver linear sections will be called the preliminary (PF) and additional (AF) filters. Amplitude–frequency responses of the PF and AF must adhere to the same law. Resonance frequencies of the PF and AF must be detuned relative to each other by a value determined from results [18,19] for the purpose of providing uncorrelated statistics at the PF and AF outputs. The detuning value between the resonance frequencies of the PF and AF is in excess of the effective signal bandwidth ΔF_a. As was shown in [18,19], if this value is run into $4 - 5\Delta F_a$, a coefficient of correlation between statistics at the PF and AF outputs tends to zero. Practically these statistics may be regarded as uncorrelated. The effective PF bandwidth is equal to signal frequency one (it may be greater but this is undesirable, as the interference power at the PF output is proportional to the effective bandwidth). The effective AF bandwidth may be smaller than that of the PF but in the present paper the effective AF bandwidth is assumed to be the same as that of the PF. By these means uncorrelative observed samples of stochastic processes are arranged at the PF and AF outputs. These samples adhere to the same distribution law with the same statistic characteristics given that the same process is present at the PF and AF inputs (even if this process is white noise having a correlation function $(N_0/2)\delta(t_2 - t_1)$).

A physicotechnical interpretation of the signal detection algorithm (12) is the following.

- AF may be a source of the reference sample $\eta_1, \ldots, \eta_N$ from the interference region Z^*. The AF resonance frequency is detuned relative to the carrier frequency of a signal by a value which may be defined based on [18,19,35–42] depending on a specific practical situation.
- PF may be a source of the observed sample $X_1, \ldots, X_N$ from the interference region Z. The PF bandwidth is in agreement with the effective signal bandwidth. The value of the PF bandwidth is in agreement with that of the AF bandwidth.
- The first term in (12) corresponds to a synthesis of the detector correlation channel with twice the gain.
- The second term in (12) corresponds to a synthesis of the detector autocorrelation channel contacted with the PF.
- The third term in (12) corresponds to a synthesis of the detector autocorrelation channel contacted with the AF.
- The statistic of the detector autocorrelation channel contacted with the PF is subtracted from the statistic of a detector autocorrelation channel contacted with the AF. As a result $\Sigma_{i=1}^N \eta_i^2 - \Sigma_{i=1}^N \xi_i^2 \Rightarrow 0$ as $N \Rightarrow \infty$ in the statistic sense.
- The statistic of the detector autocorrelation channel contacted with the PF is subtracted from the statistic of a detector correlation channel. As a result complete compensation of the noise component $\Sigma_{i=1}^N a_i^* \xi_i$ of the signal detection algorithm (4) is executed given $a_i^* = a_i$, where a_i^* is a signal model.

The detector block diagram presented in Fig. 2 has been synthesized based on the above-stated physicotechnical interpretation of the generalized signal detection algorithm (12) [21,22].

3. EXPERIMENT

Before proceeding to the results of experimental investigation of the generalized signal detection algorithm we first say some words about the synthe-

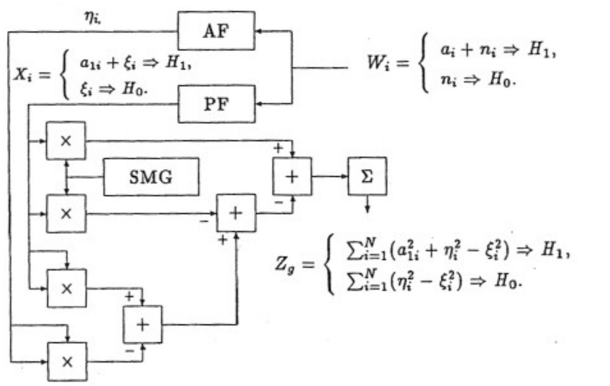

FIG. 2. Physicotechnical interpretation of the generalized signal detection algorithm.

sis of the generalized detector block diagram represented in Fig. 3. It should be stressed that the synthesis of the generalized detector block diagram has been carried out based on theoretical results [17,22–33].

A generalized detector block diagram with a threshold apparatus (THRA) has been synthesized for signal detection by the Neuman–Pierson criterion (see Fig. 3). Switch K1 takes position 1 given "no" signal in input stochastic process. At that instant statistical characteristics of background noise are defined. As was noted in [12], statistical characteristics of interferences for self-training systems can be defined by a radar guidance, for example, in a space with "no" target (signal) as is known a priori. This approach can be used for definition of statistical characteristics of background noise and threshold K_N^*, eliminating simultaneously the influence of the generalized detector correlation channel. At the first instant we must fulfill the condition $a_1(t) = a^*(t) = 0$, where $a^*(t)$ is a signal model. After definition of background noise statistical characteristics and threshold K_N^* by the decision function $\varphi(\alpha)$ (decision block, DB) switch K1 takes position 2. THRA should be taken out of service from the indicators $f_1[Z_g, t]$ and $f_2[Z_g, a^*]$ by switch K2 given that the decision "yes" signal in input stochastic process has been made performing herewith their unlocking and allowing simultaneous analysis of detector output process to define the signal structure and unknown parameters of signal (position 2 for switch K2). If THRA output is connected to the signal model generator switching apparatus (SGSA) input which can be made as a relay device there will be the possibility of automatic switching on the signal model generator (SMG) to define unknown signal parameters and structure given that the decision "yes" signal in input stochastic process has been made. At that instant we must fulfill the condition $a^*(t) \neq 0$ or SMG must be switched on. We will see in what follows how the condition $a_1(t) = a^*(t)$ must be fulfilled. It should be noted that the generalized

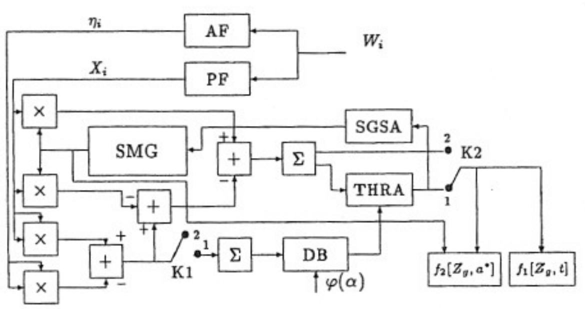

FIG. 3. Generalized detector.

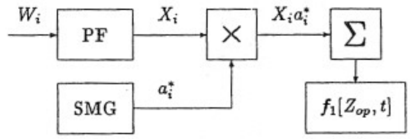

FIG. 4. Correlation detector.

detector can be realized by a widely known element base and it is not required to create a new one. One possibility for practical realization of the generalized detector is represented in Fig. 3. It is reasonable that there are many practical realizations of the generalized detector but the development concept of all variants is the same.

3.1. Conditions of Experimental Investigations

- Noise has been imitated by a generator of random uncorrelated numbers which adhere to the normal distribution law with zero mean and finite variance. Conditions of PF and AF choice and correlation between PF and AF amplitude–frequency responses have been discussed in detail in [22–25].
- Signal is a zero phase-manipulated sequence of the recurrent code with a number of the first digits to 5 and period N ($N = 31$).
- Averaged process has been observed at the detector outputs:

—The optimal detector,

$$Z_{op} = \frac{1}{N} \sum_{i=L-N}^{L} X_i a_i^*,$$

a block diagram of which is represented in Fig. 4.

—The generalized detector,

$$Z_g = \frac{1}{N} \left\{ \sum_{i=L-N}^{L} 2X_i\, a_i^* - \sum_{i=L-N}^{L} X_i^2 + \sum_{i=L-N}^{L} \eta_i^2 \right\},$$

where

$$X_i = \begin{cases} a_{1i} + \xi_i \Rightarrow H_1 \\ \xi_i \Rightarrow H_0 \end{cases},$$

is a process at the PF output; a_{1i} is a signal. When experimental investigations are performed a signal location on the time axis has been determined as $a_{1i} = a_1$ given $L = L_0 + N$ and $a_{1i} = 0$ given $L \neq L_0 + N$. L_0 is an instant of the origin. ξ_i is noise at the PF output. a_i^* is a signal model. Given $L = L_0 + N$, a signal model formed in a receiver is matched with signal by structure within the limits of the time interval. η_i is noise at the AF output. Sequences ξ_i

and η_i are not correlated due to choosing of PF and AF amplitude–frequency responses [22,30].

- It was suggested that the energy of the signal model for the generalized detector has been changed from zero to a definite fixed value which is considerably in excess of the signal energy for the optimal detector. Variation of signal model energy has been carried out by changing signal model amplitude (the element of tracking system).
- Sample size is 60 for each differential time change.
- The process at the detector output has been performed in the following coordinate systems:
  —the optimal detector, with the indicator $f_1[Z_{op}, t]$ where t_i is discrete time;
  —the generalized detector, with the indicators $f_1[Z_g, t]$ and $f_2[Z_g, a^*]$ simultaneously.
- Both detectors have been held under absolutely equal experimental conditions in terms of energy parameters of signal and noise at the detector input.
- A tracking window procedure has been used for signal detection.
- Experimental investigations have been carried out for the following signal-to-noise ratios by power at the optimal and generalized detector inputs (the PF output):
  —15.92 dB is a region of clear signal detection by the optimal detector;
  —0.96 dB is a region of failure to detect the signal by the optimal detector.

4. RESULTS

4.1. Signal-to-Noise by Power is Equal to 15.92 dB

Optimal Detector

Given a "yes" signal in the input stochastic process the statistic Z_{op} at the optimal detector output observed by the indicator $f_1[Z_{op}, t]$ is represented in Fig. 5. Signal-to-noise by power at the optimal detector input is equal to 15.92 dB. The abscissa is a time axis. Statistic Z_{op} is observed along the abscissa. The ordinate is the amplitude of the statistic Z_{op} at the optimal detector output. During one differential time change the sample size is 60.

Referring to Fig. 5, it can be seen that the noise component of the optimal detector is observed at the differential time changes from 0 to 29 and from 31 to 68. The noise component

$$Z_{op} = \frac{1}{N} \sum_{i=L-N}^{L} a_i^* \xi_i \tag{13}$$

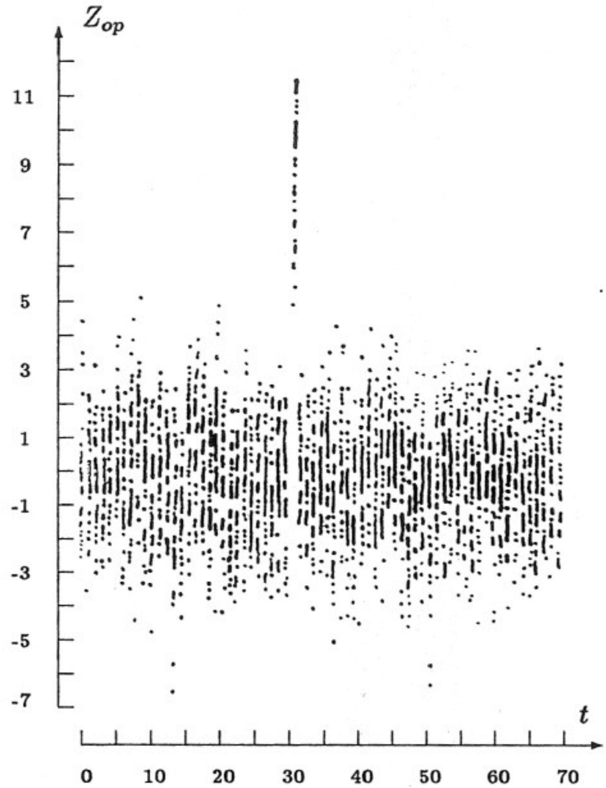

FIG. 5. Optimal detector. Signal-to-noise by power is equal to 15.92 dB.

caused by interaction between the signal model and the noise has zero mean and variance 1.47. The process

$$Z_{op} = \frac{1}{N} \sum_{i=L-N}^{L} \{a_{1i} a_i^* + a_i^* \xi_i\} \tag{14}$$

observed at the 30th differential time change indicates a "yes" signal in the input stochastic process. Here $\frac{1}{N} \sum_{i=L-N}^{L} a_{1i} a_i^*$ is an average of signal energy given $a^* = a_1$. Signal-to-noise by power at the detector output is equal to 15.74 dB. Root-mean-square scatter of points relative to an average of signal energy (Fig. 5) is caused by variance of noise component. Signal is clearly detected by the optimal detector given that signal-to-noise by power at the detector input is equal to 15.92 dB.

Generalized Detector: Observation by Indicator $f_1[Z_g, t]$

Background noise

$$Z_g = \frac{1}{N} \sum_{i=L-N}^{L} \{\eta_i^2 - \xi_i^2\} \tag{15}$$

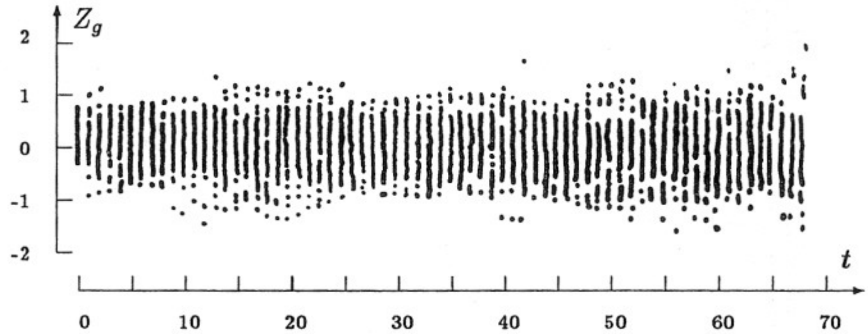

FIG. 6. Generalized detector background noise.

characterized by zero mean and variance 0.64 (Fig. 6) is observed at the detector output given $a^* = 0$ (the SMG must be switched off) and $a_1 = 0$ ("no" signal in the input stochastic process). It is easy to verify that variance of background noise is less than that of the noise component of the optimal detector (see Fig. 5). Here it is pertinent that the variance of the noise component is defined by the product of signal model energy and variance of noise ξ at the PF output. With an increase in the energy of the signal model the variance of the noise component increases. Variance of background noise is independent of signal model energy and average of signal energy. The statistic of background noise is represented in Fig. 6.

The statistic obtained given $a^* = 0, a_1 \neq 0$ is represented in Fig. 7. These conditions imply that the SMG is switched off and there is a "yes" signal in the input stochastic process. The statistic observed at the generalized detector output under these conditions is described by the expression

$$Z_g = \frac{1}{N} \sum_{i=L-N}^{L} \{-a_{1i}^2 - 2a_{1i}\xi_i - \xi_i^2 + \eta_i^2\}. \quad (16)$$

An average of signal energy (the first term) and a random component $\frac{1}{N}\sum_{i=L-N}^{L} 2a_{1i}\xi_i$ caused by interaction among signal and noise (the second term) is added with the "minus" sign to background noise. An average of the signal energy and a random component are present within the limits of the signal interval on the time axis solely. Variance of the random component is maximum given $L = L_0 + N$. Comparing Figs. 6 and 7 one can readily see as the statistic Z_g at the detector output is varied given a "yes" signal in the input stochastic process. It is easily seen that the statistic Z_g at the detector output is completely matched with the background noise on the time axis region given a "no" signal in the input stochastic process (see Figs. 6 and 7, differential changes from 60 to 68). Analyzing the statistic represented in Fig. 7, it may be deduced that a signal is clearly detected in the input stochastic process; moreover the signal may be an arbitrary structure. Variance of the statistic Z_g at the detector output rises steeply in a distinct region of the time axis. This phenomenon is caused by interaction between signal and noise. We can state with assurance there is a "yes" signal in input stochastic process. But the signal structure is unknown and it is unjustified to

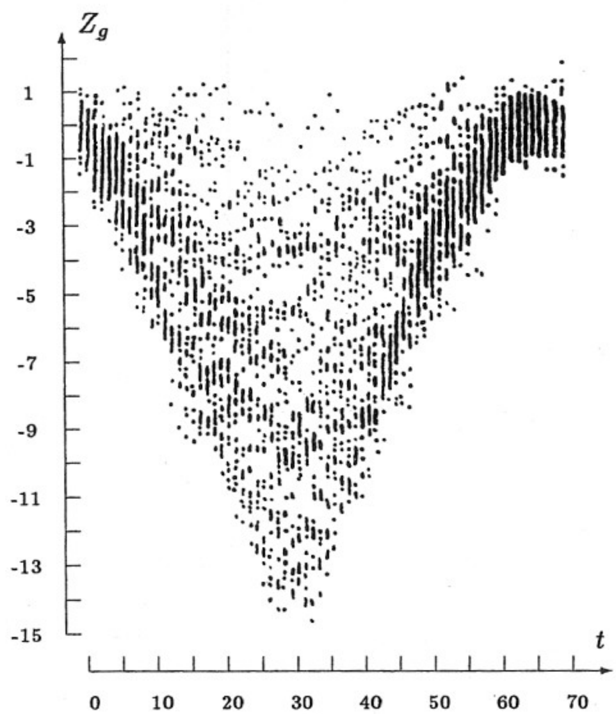

FIG. 7. Generalized detector. Autocorrelation channel; indicator $f_1[Z_g, t]$.

reason that the detected signal has an expected structure.

The statistic

$$Z_g = \frac{1}{N}\sum_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2 + 2a_i^*\xi_i - 2a_{1i}\xi_i - \xi_i^2 + \eta_i^2\} \qquad (17)$$

is observed at the detector output given $a^* \neq 0$ (the SMG is switched on for definition of signal structure and parameters) and $a_1 \neq 0$ ("yes" signal in input stochastic process). Signal structure is defined given that the signal model is matched with a signal. Given $a^* = a_1$ and $L \neq L_0 + N$, which means no falling within the limits of the same time interval between signal model and signal, or $L = L_0 + N$, which means falling within the limits of the same time interval between signal model and signal, the statistic at the generalized detector output is represented in Fig. 8. In this connection it is necessary to consider two cases.

Case $L \neq L_0 + N$. In this case due attention should be given to observation sections of the time axis where signal influence is present (the differential time changes 0–60) and absent (the differential time changes 61–68). Given "no" signal the statistic at the detector output analyzed within the limits of the time interval is determined by the

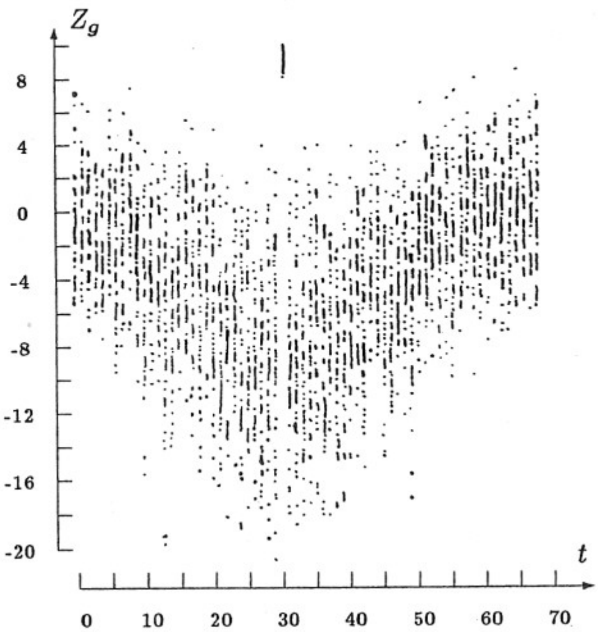

FIG. 8. Generalized detector. Correlation and autocorrelation channels; indicator $f_1[Z_g, t]$.

expression

$$Z_g = \frac{1}{N}\sum_{i=L-N}^{L}\{2a_i^*\xi_i - \xi_i^2 + \eta_i^2\}. \qquad (18)$$

The noise component caused by interaction between signal model and noise at the PF output is added to the background noise. The variance of this statistic is increased in comparison with that of background noise because of the increase in the degree of uncertainty. This is the reason it is expedient to carry out an analysis of the statistic at the detector output under signal detection (but not definition of signal structure and parameters) given that the SMG is switched off ($a^* = 0$) for a decrease in the degree of uncertainty. This is an important and essential salient feature of generalized detector operation. The identification process suggests a signal structure definition: the signal of an expected structure or some other signal is detected.

The statistic

$$Z'_g = \frac{1}{N}\sum_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2 + 2a_i\xi_i - 2a_{1i}\xi_i\} \qquad (19)$$

is added to background noise within the limits of the time interval given a "yes" signal. It is easy to notice a drastic increase in the variance of the statistic as a result of the influence of the first and second terms because of noncorrelation between a_1 and a^* and a great amplitude of signal, but the variance of the statistic is independent of these terms. However, the third and fourth terms presenting the correlation channel noise component and the autocorrelation channel random component of the generalized detector influence the variance of the statistic, as they are stochastic. A further problem is to define whether essential variation of the statistic's variance is caused by a "yes" signal of expected structure in input stochastic process. To solve this problem it is necessary to define a signal structure and parameters falling within the limits of the same time interval between signal model and signal.

Case $L = L_0 + N$. It is reasonable to consider this case at the differential time change 30 (see Fig. 8). Here the statistic at the detector output is determined by the expression

$$Z_g = \frac{1}{N}\sum_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2 - \xi_i^2 + \eta_i^2\} \qquad (20)$$

as a_1 and a^* appear to be correlated given $L = L_0 +$

N, and the correlation channel noise component and autocorrelation channel random component of the generalized detector are completely compensated at this differential time change by correlation between them whereas they are not correlated at other differential time changes within the limits of the time axis. The detected signal is clearly seen at the differential time change 30. Signal-to-noise by power at the detector output is equal to 22.98 dB.

A distinctive property of the differential time change 30 (see Fig. 8) is a severe decrease in variance of statistic at the detector output by value and an abrupt jump up to the average of signal energy. The variance of the signal component relative to an average of signal energy is determined by the variance of background noise (15) (see Fig. 6, the differential time change 30). Mentioned features define the detection of a signal with the expected structure and signal location within the limits of the time axis. At this differential time change the position of the statistic relative to the zero axis is characterized by a mean corresponding to the average of signal energy and variance of background noise.

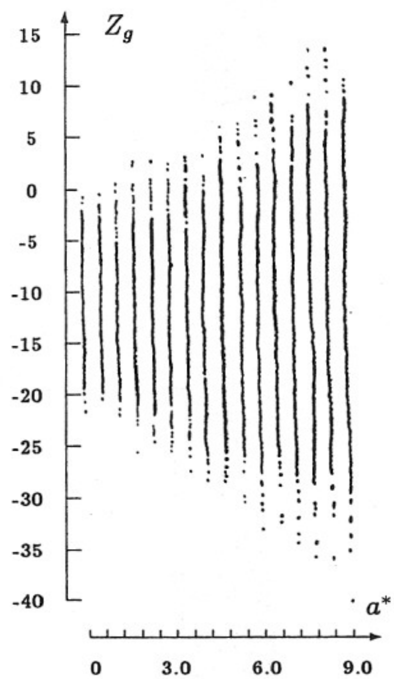

FIG. 9. Generalized detector. No falling within the limits of the same time interval between signal model and signal by structure; indicator $f_2[Z_g, a^*]$.

Generalized Detector: Observation by Indicator $f_2[Z_g, a^*]$

The indicator $f_2[Z_g, a^*]$ is necessary for the following reason. For the optimal detector it is not necessary to fulfill the condition $a^* = a_1$, as the signal amplitude, as a rule, is unknown. The amplitude of the signal model (reference voltage) is chosen given $a^* = ka_1$ where k is a proportionality coefficient which can be either greater or lesser than one in the general case. For the generalized detector a compensation among the correlation channel noise component and the autocorrelation channel random component is carried out given $a^* = a_1$ ($k = 1$) solely. Additional information at both signal detection and definition of signal parameters can be gleaned by the indicator $f_2[Z_g, a^*]$.

Given $L \neq L_0 + N$ the statistic at the detector output as a function of variation of the signal model amplitude is represented in Fig. 9. The tracking window of the correlation channel is mismatched with the signal within the limits of the time axis. One can see that the statistic at the generalized detector output is displaced downward by a value E'_{a1} relative to the zero axis given that the SMG is switched off ($a^* = 0$).

E'_{a1} is a part of the total signal energy which is equivalent to the matching degree of the tracking window with the signal within the limits of the time axis. Given $a^* = 0$ the variance of the statistic is determined by the stochastic process

$$Z''_g = \frac{1}{N}\sum_{i=L-N}^{L} \{-2a_{1i}\xi_i - \xi_i^2 + \eta_i^2\}. \tag{21}$$

With an increase in the amplitude of the signal model the variance of the statistic at the generalized detector output increases linearly due to the additional action of the correlation channel noise component $\frac{1}{N}\Sigma_{i=L-N}^{L} 2a_i^*\xi_i$ which increases the degree of uncertainty.

Given $L = L_0 + N$ the statistic at the generalized detector output is represented as a function of the signal model amplitude in Fig. 10. The tracking window is matched with the signal within the limits of the time axis. Given $a^* = 0$ (the SMG is switched off) the parameters of the statistic at the generalized detector output correspond to those at the differential time change 30 in Fig. 7. Herewith the observed statistic is displaced downward by an average of signal energy E_{a1} relative to the zero axis of coordinate system of the indicator $f_2[Z_g, a^*]$. With an increase in the amplitude of signal model the mean of the statistic at the generalized detector output is varied by the expression $\frac{1}{N}\Sigma_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2\}$ and has a positive tilt angle relative to the zero axis of the

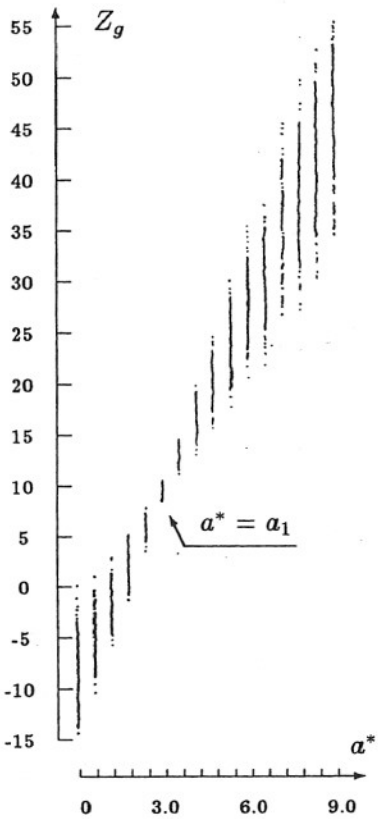

FIG. 10. Generalized detector. Falling within the limits of the same time interval between signal model and signal by structure; indicator $f_2[Z_g, a^*]$.

coordinate system of the indicator $f_2[Z_g, a^*]$. Given $a^* = 0.5a_1$ the mean is equal to zero. Given $a^* = a_1$ the mean is an average of signal energy. With further increase in the amplitude of signal model the mean increases linearly. Variance of the statistic at the detector output is determined by the stochastic process

$$Z'''_g = \frac{1}{N} \sum_{i=L-N}^{L} \{2a_i^* \xi_i - 2a_{1i}\xi_i - \xi_i^2 + \eta_i^2\}. \quad (22)$$

With an increase in the amplitude of the signal model from 0 to $a^* = a_1$ the variance of the statistic at the detector output decreases. Given $a^* = a_1$ the variance is minimum and corresponds to that of background noise. By this means, the effect of complete compensation between the correlation channel noise component $\frac{1}{N}\Sigma_{i=L-N}^{L} 2a_i^* \xi_i$ caused by interaction among signal model and noise and the autocorrelation channel random component $\frac{1}{N}\Sigma_{i=L-N}^{L} 2a_{1i}\xi_i$ caused by interaction among signal and noise is realized owing to their correlation. In doing this, variance of the signal component is caused by variance of the background noise at the differential time change 30 (see Figs. 6 and 8).

With a further increase in the amplitude of the signal model the variance of the statistic at the detector output increases. By this means, with an increase in the amplitude of the signal model from 0 to $a^* = a_1$ the variance of the statistic at the detector output decreases to a minimum. Given $a^* > a_1$ the variance of the statistic at the detector output increases. Furthermore the symmetry axis of the statistic at the detector output determined by the expression $\frac{1}{N}\Sigma_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2\}$ can easily been seen. By this means, given $L = L_0 + N$, the differential time change 30 in Fig. 8 and the equality $a^* = a_1$ in Fig. 10 define a signal location on the time axis and an average of signal energy.

Analysis of experimental results given that signal-to-noise by power at the detector input is equal to 15.92 dB leads us to the following conclusions.

- Signal is detected by employment of the optimal detector given that signal-to-noise by power at the detector input is equal to 15.92 dB. Signal-to-noise by power at the optimal detector output is equal to 15.74 dB.
- Evidence of signal detection by employment of the optimal detector is given by the fact that the signal component exceeds the noise component caused by the interaction between signal model and noise. This corresponds to the criterion of likelihood function maximum.
- Signal is detected by employment of the generalized detector given that signal-to-noise by power at the detector input is equal to 15.92 dB. Signal-to-noise by power at the generalized detector output is equal to 22.98 dB.
- Evidence of signal detection by employment of the generalized detector is the following:

—Excess of signal component over background noise given $a^* = 0$ (the SMG is switched off). This corresponds to the criterion which can be defined by the likelihood function. Given "no" signal in input stochastic process ($a_1 = 0$) the excess is absent and background noise is observed solely. Displacement of statistic at the detector output is caused by sign "minus" of autocorrelation channel of the generalized detector (see Fig. 7).

—Given $a^* = 0$ the statistic at the detector output is displaced downward by a value of signal energy. Under mismatching between the tracking window of correlation channel and signal on the time axis a displacement of statistic at the detector output is carried out by a value of signal energy part which is equivalent to matching degree among the tracking window of correlation channel and signal on the time axis (see Fig. 9). Given that the tracking window of correlation channel is completely matched with signal on the time axis a displacement of statistic at the detector output is carried out by average of signal energy (see Fig. 10).

—Under incomplete matching between the tracking window of the correlation channel and the signal on the time axis the variance of the statistic at the generalized detector output increases relative to a symmetry axis determined by signal energy with increase in the amplitude of signal model. In this case the signal energy is equivalent to matching degree among the tracking window of correlation channel and signal on the time axis. Given "no" signal in the input stochastic process, or the tracking window of correlation channel being completely mismatched with the signal on the time axis, the variance of the statistic at the generalized detector output increases with increase in the amplitude of signal model relative to a symmetry axis corresponding to the zero axis. Given that the tracking window of the correlation channel is completely matched with the signal on the time axis, the variance of the statistic at the generalized detector output decreases with increase in the amplitude of signal model from the zero to amplitude of signal. Given equality between amplitude of signal model and amplitude of signal the variance of the statistic at the generalized detector output becomes minimum and corresponds to the variance of background noise (see Figs. 6, 8, and 10, accordingly). With a further increase in the amplitude of signal model the variance of the statistic at the generalized detector output increases. Given that the amplitude of signal model is equal to the amplitude of signal an effect of complete compensation among correlation channel noise component $\frac{1}{N}\Sigma_{i=L-N}^{L} 2a_i^* \xi_i$ caused by interaction between signal model and noise and autocorrelation channel random component $\frac{1}{N}\Sigma_{i=L-N}^{L} 2a_{1i}\xi_i$ caused by interaction between signal and noise is observed by virtue of their correlation given that the tracking window of correlation channel is completely matched with signal on the time axis.

—If the amplitude of signal model is varied the statistic at the generalized detector output has a symmetry axis observed by the coordinate system of the indicator $f_2[Z_g, a^*]$ given a complete match between the tracking window of the correlation channel and the signal on the time axis. Symmetry axis is determined by the expression $\frac{1}{N}\Sigma_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2\}$. The presence of a symmetry axis tilt angle relative to abscissa of coordinate system of the indicator $f_2[Z_g, a^*]$ (axis a^*) is a marker "yes" signal in the input stochastic process (see Fig. 10).

- The presence of a signal component given equality between amplitude of signal model and the amplitude of signal and decrease in variance of the statistic at the generalized detector output from a value determined by (22) to variance of background noise (in other words, a decrease in the degree of uncertainty to minimum) are a consequence of the above-mentioned evidence of signal detection by employment of the generalized detector (see Fig. 8, the differential time change 30).
- Generalized detector has the greater informative evidence in comparison with optimal detection under signal detection and definition of signal parameters. Signal detection, definition, and estimation of signal parameters are carried out by the indicators $f_1[Z_g, t]$ and $f_2[Z_g, a^*]$ simultaneously for the generalized detector.

4.2. Signal-to-Noise by Power Is Equal to 0.96 dB

Optimal Detector

Given "no" signal in input stochastic process the statistic at the detector output is represented in Fig. 11. Presented process is noise component (13) caused by interaction between signal model and noise. The statistic has zero mean. Variance of the statistic is approximately equal to 0.23. Given "yes" signal in input stochastic process the statistic at the detector output is represented in Fig. 12. This statistic is determined by expression (14). Signal-to-noise by power at the detector output is equal to 0.75 dB.

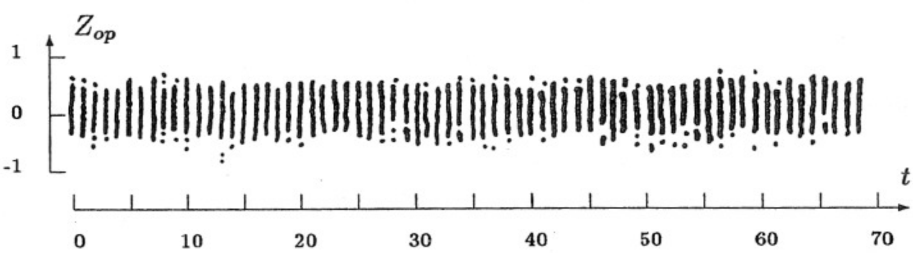

FIG. 11. Optimal detector. "No" signal.

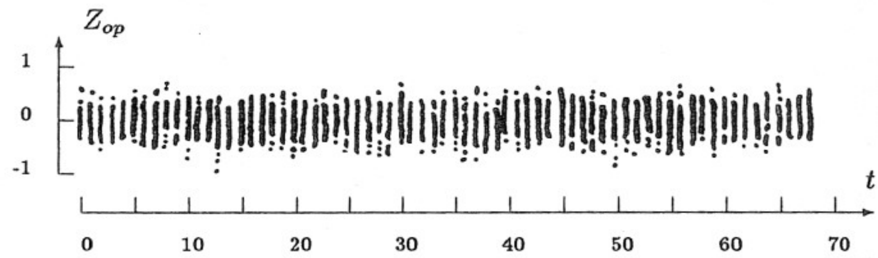

FIG. 12. Optimal detector. "Yes" signal.

Comparing results represented in Figs. 11 and 12 it can be said with assurance that the statistics at the detector output given "no" and "yes" signal are no different. Signal-to-noise by power 0.96 dB is a region of the failure to detect a signal by the optimal detector.

Generalized Detector: Observation by Indicator $f_1[Z_g, t]$

Background noise (15) is observed at the detector output given $a^* = 0$ (the SMG is switched off) and $a_1 = 0$ ("no" signal in input stochastic process). It is reasonable to point out that variance of noise component of the optimal detector is determined by product among signal model energy and noise variance. With an increase in energy of signal model the variance of the noise component increases and consequently a variance of statistic at the optimal detector output increases. Background noise is independent of both the signal model energy and the signal energy.

Given "yes" signal in input stochastic process ($a_1 \neq 0$) and SMG switched off ($a^* = 0$) the statistic at generalized detector output is represented in Fig. 13. This statistic is determined by expression (16). An average of signal energy (the first term) and random component caused by interaction between signal and noise (the second term) is added to background noise with sign "minus." It is interior to the time interval where this is a "yes" signal solely. Variance of the statistic at the detector output is maximum given $L = L_0 + N$. But it is obvious for powerful signals (see the previous subsection).

Comparing Figs. 6 and 13 one can see evidence of signal detection as variance of the statistic at the detector output is greater than that of background noise. Complete agreement with variance of background noise is observed given "no" signal on the time axis (the differential time changes 61–68, Fig. 13). Analysis of the statistic presented in Fig. 13 allows definite conclusion "yes" signal in input stochastic process; moreover signal may be arbitrary structure. An increase in variance of the statistic at the detector output indicates an interaction between signal and noise that allows definite conclusion: "yes" signal in input stochastic process. But in this case the problem is not concerned with signal structure and exact definition of signal parameters.

Given that SMG is switched on for definition of structure and parameters of detected signal ($a^* \neq 0$) and "yes" signal in input stochastic process ($a_1 \neq 0$) the statistic at the detector output is determined by expression (17). Given $a^* = a_1$ and $L \neq L_0 + N$ (no falling within the limits of the same time interval between signal model and signal) or $L = L_0 + N$ (falling within the limits of the same time interval between signal model and signal) the statistic at the

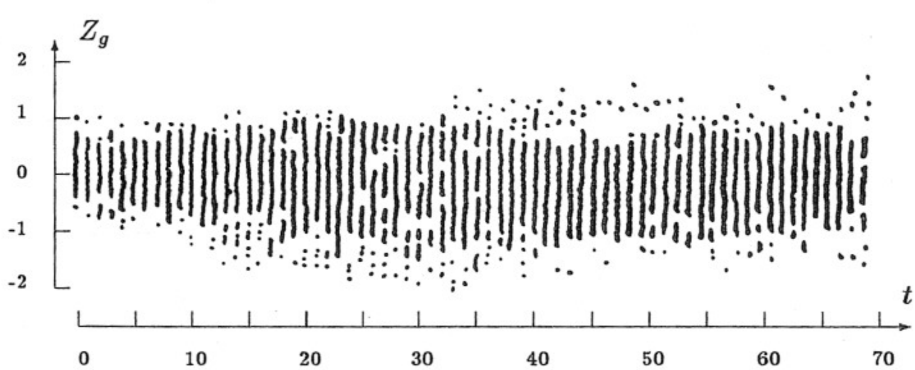

FIG. 13. Generalized detector. Autocorrelation channel; indicator $f_1[Z_g, t]$.

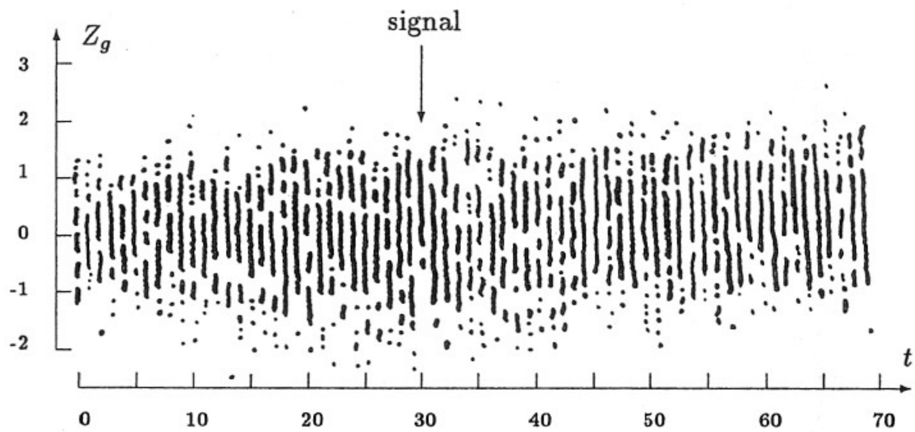

FIG. 14. Generalized detector. Correlation and autocorrelation channels; indicator $f_1[Z_g, t]$.

generalized detector output is represented in Fig. 14. It is necessary to consider two cases.

Case $L \neq L_0 + N$. Given "no" signal within the limits of the time interval (the differential time changes 61–68) the statistic at the detector output is determined by expression (18). The noise component of the correlation channel defined by interaction among signal model and noise is added to background noise. This leads to increased variance in comparison with that of background noise as the degree of uncertainty rises. Because of this, it is expedient to carry out signal detection (not definition of signal structure and parameters) given that the SMG is switched off ($a^* = 0$) for the purpose of decreasing the degree of uncertainty. This is important and essential salient evidence of generalized detector operation.

Process (19) is added to background noise within the limits of the time interval with "yes" signal on the time axis (the differential time changes 0–60). It is easy to notice an increase in variance of the observed statistic as a result of the influence of the first and second terms (19) because of noncorrelation among a_1 and a^*. The third and fourth terms (19) present the influence of the correlation channel noise component and the autocorrelation channel random component of the generalized detector on the variance of the statistic at the detector output as they are stochastic.

Case $L = L_0 + N$. It is reasonable to consider this case at the differential time change 30 (see Fig. 14). At this moment the statistic at the detector output is determined by expression (20) because a_1 and a^* appear to be correlated given $L = L_0 + N$. Correlation channel noise component and autocorrelation channel random component are exactly compensated at this moment by correlation between them whereas these components are noncorrelated at other differential time changes on the time axis. Signal is clearly detected at differential time change 30. A decrease in variance of the statistic at the detector output is specific evidence for the differential time change 30 in Fig. 14. This is a characteristic singularity of the generalized detector for weak signals. Incidentally, the variance of signal component with respect to an average of signal energy is determined by variance of background noise (15) (see Fig. 6). Indicated evidences define a signal location on the time axis besides a detection of the expected signal. At differential time change 30 the position of the statistic at the detector output relative to the zero axis is characterized by a mean corresponding to an average of signal energy and by the variance of background noise.

Generalized Detector: Observation by Indicator $f_2[Z_g, a^*]$

Given $L \neq L_0 + N$ the statistic at the generalized detector output as a function of signal model amplitude is represented in Fig. 15. Tracking window of correlation channel is mismatched with signal on the time axis. One can see that the statistic is shifted down on value E'_{a1} relative to the zero axis given the SMG is switched off ($a^* = 0$). E'_{a1} is a part of signal energy, which is equivalent to the matching degree of the tracking window with signal on the time axis. Given $a^* = 0$ the variance of the statistic is determined as the variance of the stochastic process (21).

With increase in the amplitude of signal model the variance of the statistic at the generalized detector output increases linearly due to an additional action of correlation channel noise component $\frac{1}{N}\Sigma_{i=L-N}^{L} 2a_i^* \xi_i$, which increases the degree of uncertainty. Given $L = L_0 + N$, the statistic at the generalized detector output as a function of signal model amplitude is represented in Fig. 16. The tracking window of the correlation channel is matched with the signal on the time axis. Given that $a^* = 0$ parameters of the statistic at the detector output correspond to parameters at differential time change 30 (see Fig. 13). Herewith the statistic at the generalized detector output is down displaced an average of signal energy E_{a1} relative to the zero axis. With an increase in the amplitude of the signal model the mean of the statistic at the detector output is varied by the expression $\frac{1}{N}\Sigma_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2\}$ and has a positive tilt angle relative to the zero axis. Given $a^* = a_1$ the mean is an average of signal energy. With further increase in the amplitude of the signal model the mean increases linearly. Variance of the statistic at the generalized detector output is determined by variance of the total background noise (22). With increase in the amplitude of the signal model from $a^* = 0$ to $a^* = a_1$ the variance of the statistic at the generalized detector output decreases. Given $a^* = a_1$ this variance is minimum and equal to background noise one. By this means, the effect of complete compensation between the correlation channel noise component $\frac{1}{N}\Sigma_{i=L-N}^{L}2a_i^*\xi_i$ and the autocorrelation channel random component $\frac{1}{N}\Sigma_{i=L-N}^{L}2a_{1i}\,\xi_i$ is realized owing to their correlation.

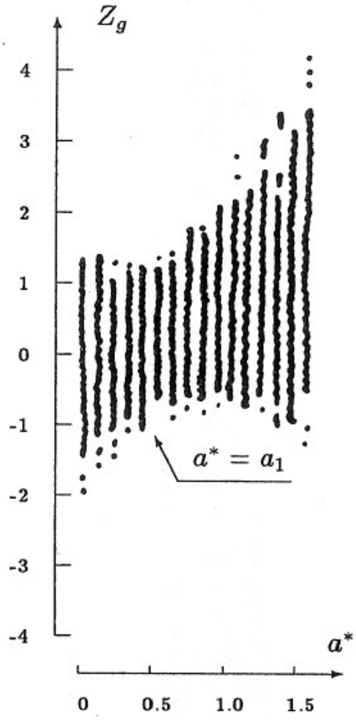

FIG. 16. Generalized detector. Falling within the limits of the same time interval between signal model and signal by structure; indicator $f_2[Z_g, a^*]$.

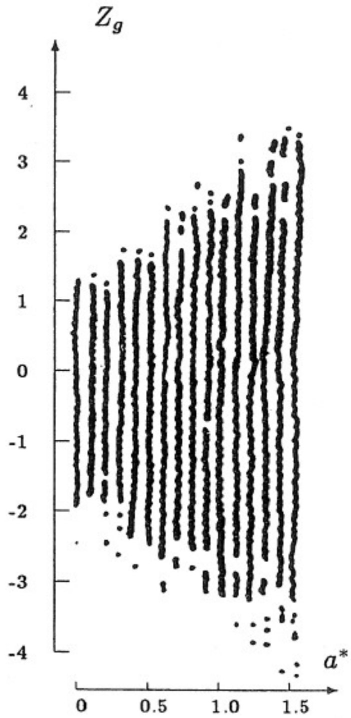

FIG. 15. Generalized detector. No falling within the limits of the same time interval between signal model and signal by structure; indicator $f_2[Z_g, a^*]$.

Variance of signal component is caused by background noise one (15) at the differential time change 30 (see Figs. 6 and 14). With further increase in the amplitude of the signal model the variance of the statistic at the generalized detector output increases.

By this means, with an increase in the amplitude of signal model from 0 to $a^* = a_1$ the variance of the statistic at the generalized detector output decreases to a minimum. Given $a^* > a_1$ one can see an increase in the variance of the statistic at the generalized detector output. Furthermore the symmetry axis of the statistic at the generalized detector output determined by the expression $\frac{1}{N}\Sigma_{i=L-N}^{L}\{2a_{1i}a_i^* - a_{1i}^2\}$ is clearly defined. Given $L = L_0 + N$, the differential time change 30 in Fig. 14, and the equality $a^* = a_1$ in Fig. 16, define a signal location on the time axis and an average of signal energy.

Analysis of experimental investigations for weak signals leads us to the following conclusions.

- Signal is not detected using the optimal detector given signal-to-noise by power at the detector input equal to 0.96 dB. Signal-to-noise by power at the detector output is equal to 0.75 dB.
- Signal is clearly detected using the generalized detector given signal-to-noise by power at the detector input equal to 0.96 dB. Signal-to-noise by power at the detector output is equal to −8.18 dB. This

deterioration of signal-to-noise by power at the generalized detector output is caused by interference action η. It is characteristic under detection of weak signals using the generalized detector. Theoretical results verify this fact [28,30,38–42].

• The generalized detector has the greater amount of informative evidence under signal detection and estimation of signal parameters in comparison with the optimal detector. Signal detection, definition, and estimation of signal parameters are carried out by the indicators $f_1[Z_g, t]$ and $f_2[Z_g, a^*]$ simultaneously using the generalized detector.

• Evidence of signal detection, definition, and estimation of signal parameters using the generalized detector is

—Variance of the statistic at the generalized detector output is greater than variance of background noise (the indicator $f_1[Z_g, t]$, Fig. 6).

—Variance of the statistic at the generalized detector output shrinks both above and below the instant of falling within the limits of the same time interval between signal model and signal under signal scanning on the time interval. Lower variance diminishing is more graphically evidence (the indicator $f_1[Z_g, t]$, Fig. 14, the differential time change 30).

—Given $a^* = 0$ the statistic at the generalized detector output is lower relative to the zero axis (the indicator $f_2[Z_g, a^*]$, Fig. 16).

—There is a positive tilt angle between the symmetry axis of the statistic at the generalized detector output and the zero axis of the coordinate system of the indicator $f_2[Z_g, a^*]$. This positive tilt angle is constant with increase in the amplitude of the signal model (the indicator $f_2[Z_g, a^*]$, Fig. 16).

—Variance of the statistic at the generalized detector output decreases relative to the positive tilt angle given the amplitude of signal model is varied from $a^* = 0$ to $a^* = a_1$. Given $a^* > a_1$ the variance of the statistic at the generalized detector output increases relative to the positive tilt angle (the indicator $f_2[Z_g, a^*]$, Fig. 16).

• Falling within the limits of the same time interval between signal model and signal defines a signal location on the time interval (the indicator $f_1[Z_g, t]$, Fig. 14, differential time change 30). Minimum of variance of statistic at the differential time change 30 is equal to that of background noise. Energy parameters of signal are determined by this fact.

• Evidence of the failure to detect the signal using the generalized detector is provided by the following:

—Variance of the statistic at the generalized detector output is equal to that of background noise (the indicator $f_1[Z_g, t]$).

— Given $a^* > 0$ the variance of the statistic at the generalized detector output increases relative to the zero axis of the coordinate system of the indicator $f_2[Z_g, a^*]$.

5. CONCLUSIONS

Analysis of theoretical and experimental investigations leads us to the following conclusions.

• Modification of initial prerequisites of the classical signal detection theory is the following. It is supposed that there is a frequency–time region of interferences with "yes" signal and there is a frequency–time region of interferences with "no" signal that is known a priori. This modification makes it possible to perform a theoretical synthesis of the generalized signal detection algorithm. Two uncorrelated samples are used. One of the two is a reference sample as it is known a priori that there is "no" signal in this sample. This fact allows us to obtain a jointly sufficient statistics of mean and variance of the likelihood function (or functional). The optimal signal detection algorithm of the classical theory allows to obtain only the sufficient statistic of the mean and is a component of the generalized signal detection algorithm.

• Physicotechnical interpretation of the generalized signal detection algorithm is a composition combination of correlation and autocorrelation detectors. AF is a source of reference sample. Resonance frequency of the AF is detuned relative to that of the PF. Value of detuning is greater than an effective spectral bandwidth of signal. Using the AF jointly with the PF furnishes background noise of the generalized detector. Background noise is a difference of energy characteristics of interferences at the PF and AF outputs. This difference tends to zero in the root-mean-square sense.

• Evidence of signal detection using the optimal detector is an excess of signal component over noise caused by interaction between signal model and noise that corresponds to the criterion of the likelihood function maximum.

• Signal is clearly detected using the optimal detector given signal-to-noise by power at the optimal detector input equal to 15.92 dB. Signal is not detected using the optimal detector given signal-to-noise by power at the optimal detector input equal to 0.96 dB.

• Signal is clearly detected using the generalized detector given signal-to-noise by power at the generalized detector input equal to 15.92 dB and 0.96 dB.

- Evidences of signal detection, definition, and estimation of signal parameters using the generalized detector are:

—excess of variance of statistic at the generalized detector output over variance of background noise (the indicator $f_1[Z_g, t]$);

— under picking up signal within the limits of the time interval when a given signal model is matched with signal by time the variance of the statistic at the generalized detector output decreases both overhead and beneath; for weak signals an decrease in beneath is more marked (the indicator $f_1[Z_g, t]$);

— the statistic at the generalized detector output is displaced downward relative to the zero axis of coordinate system of the indicator $f_2[Z_g, a^*]$ given the SMG is switched off (the indicator $f_2[Z_g, a^*]$);

—positive tilt angle between the zero axis of coordinate system of the indicator $f_2[Z_g, a^*]$ and the symmetry axis of the statistic at the generalized detector output is formed and held constant with increase in the amplitude of signal model (the indicator $f_2[Z_g, a^*]$);

—decrease in the variance of the statistic at the generalized detector output with increase in the amplitude of signal model from zero to amplitude of signal and after increase in the variance of the statistic at the generalized detector output with further increase in the amplitude of the signal model (the indicator $f_2[Z_g, a^*]$).

- Instant of matching between signal model and signal using the generalized detector characterizes a signal location on the time axis (the indicator $f_1[Z_g, t]$); minimum of variance of the statistic at the generalized detector output corresponding to that of background noise characterizes a signal energy (the indicator $f_2[Z_g, a^*]$).
- Evidences of the failure to detect the signal using the generalized detector are:

—variance of the statistic at the generalized detector output corresponds to that of background noise and is uniform (the indicator $f_1[Z_g, t]$);

— variance of the statistic at the generalized detector output increases uniformly relative to the zero axis with increase in the amplitude of signal model (the indicator $f_2[Z_g, a^*]$).

- The generalized detector has more informative evidences in comparison with the optimal detector. The generalized detector allows us to detect weak signals which are not detected by the optimal detector.

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