DOA estimation and the application in LMS GR

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Introdution

Beamforming technique is used by the array to design the beam former in such a way to maximize the power radiated towards the desired directions and to suppress the interference direction. Howe-ver, the limit of this technique is the beam should be steered base on a priori knowledge about direction of desired signal and interfe-rence signal. In most practical situations, for example, in adaptive ar-ray smart antenna for mobile communication, we don’t know infor-mation about signal direction for each user. A technique called a di-rection of arrival (DOA) estimation has been suggested to estimate directions of several signals. DOA estimation is an important techni-que under signal processing of spatial signals with known direction.

Introdution

LMS GR is a receiver that combine the LMS beamformer and generalized receiver to process the received signal when there is interferenece signal and presenting a good performance under interference cancellation. LMS algorithm is non-blind algorithm and requires a priori knowledge about the desired signal. If DOA of signals is unknown, we cannot apply the LMS beamformer because a model signal is also unknown. In this case, the blind beamforming algorithm may be applied, but this approach is very complex.

Our research focuses on trade-off between cancellation of interferences by LMS GR without using other beamforming algorithms based on DOA estimation when the direction of signals are unknown.

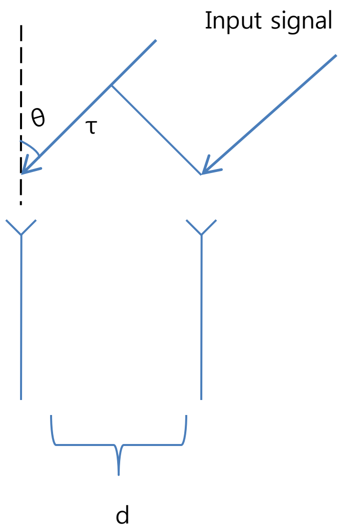
DOA Estimation

This technique allows us to estimate DOA of signals based on information about array response in the received signal. For the past few decades, a wide variety of techniques have been proposed for the DOA estimation. The subspace algorithms such as Multiple Signal Classification (MUSIC) and Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT) algorithm are the most famous ones.

The main principle of DOA algorithm is the following:

For a far field signal there is a wave path difference in array antenna due to distance between each array element; this difference generates a phase difference between the received signal of each array element. The direction of received signal can be defined based on this phase difference.

DOA estimation



Main priciple of DOA estimation

d: distance of array element

θ: incident angle

τ: wave path difference

$$\tau = \frac{d \sin \theta}{c}$$

the phase difference:

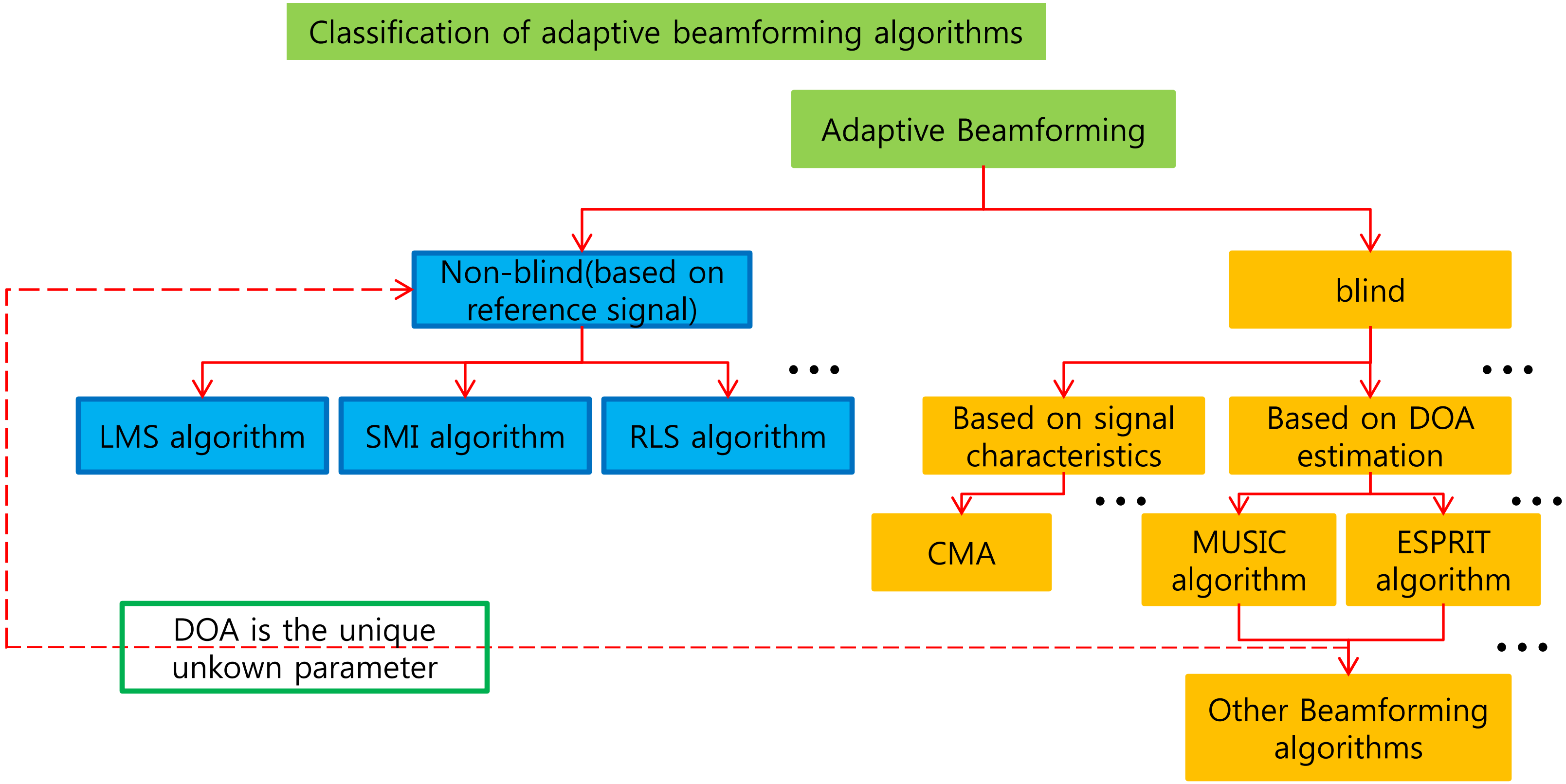
$$\psi = \exp(-jw\tau) = \exp(-jw\frac{d \sin \theta}{c})$$

c is the speed of light,

w is the signal frequency

DOA estimation and Beamforming

Adaptive beamforming and DOA estimation are the most important techniques in array signal processing. Based on various initial conditions, they can classified in the following way:



MUSIC Algorihtm

MUSIC is a high resolution technique based on exploiting the eigen-structure of input covariance matrix. MUSIC makes assumption that the noise in each channel is uncorrelated and the correlation matrix is diagonal. The incident signals are somewhat correlated generating the nondiagonal correlation matrix. If D is the number of signals, M is the number of array elements, the number of signal eigenvalues and eigenvectors is D and the number of noise eigenvalues and eigenvectors is M-D. The array correlation matrix with uncorrelated noise and equal variances is given by,

$$R_{xx} = AR_{ss}A^H + \sigma_n^2 I$$

where $A = [a(\theta_1)\ a(\theta_2) \cdots a(\theta_D)]$ is the M×D array steering matrix;

$R_{ss} = [s_1(t)\ s_2(t) \cdots s_D(t)]^T$ is D×D the signal correlation matrix;

σ_n^2 is the noise variance.

MUSIC Algorihtm

R_{xx} has the D eigenvectors associated with signals and M-D eigenvectors associated with the noise. We can then construct the M×(M-D) noise subspace spanned by the noise eigenvectors using eigenvalue ecomposition such that

$$V_N = [V_1 \, V_2 \cdots V_{M-D}]$$

<u>The noise subspace eigenvectors are orthogonal to array steering vectors at the angles of arrivals θ_D</u> and the MUSIC Pseudospectrum is given as

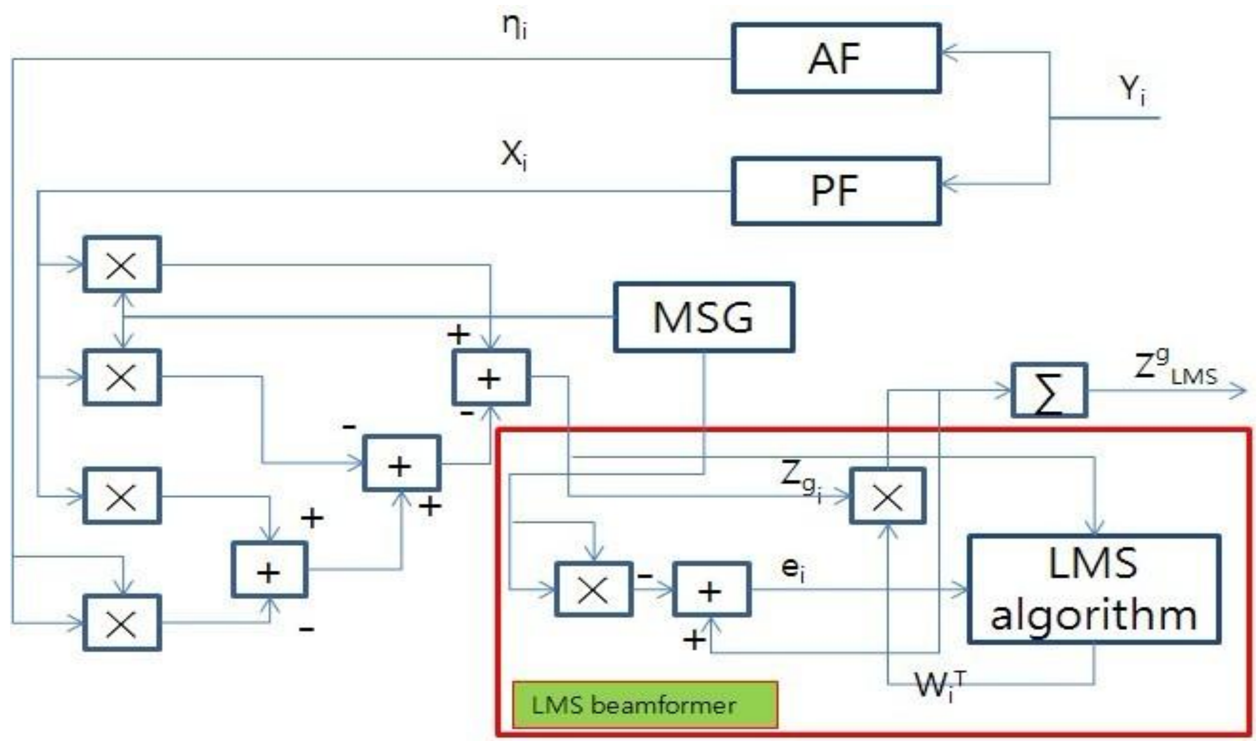
$$a(\theta) V_N = 0 \quad ideal \;\; case$$

$$P_{MUSIC}(\theta) = 1 / abs(a(\theta)^H V_N V_N^H a(\theta))$$

Thus, we can estimate the signal direction of arrival searching the spectrum peak.

LMS GR and DOA estimation

LMS GR is a combination of the LMS beamformer and generalized receiver processing the received signal with interference.

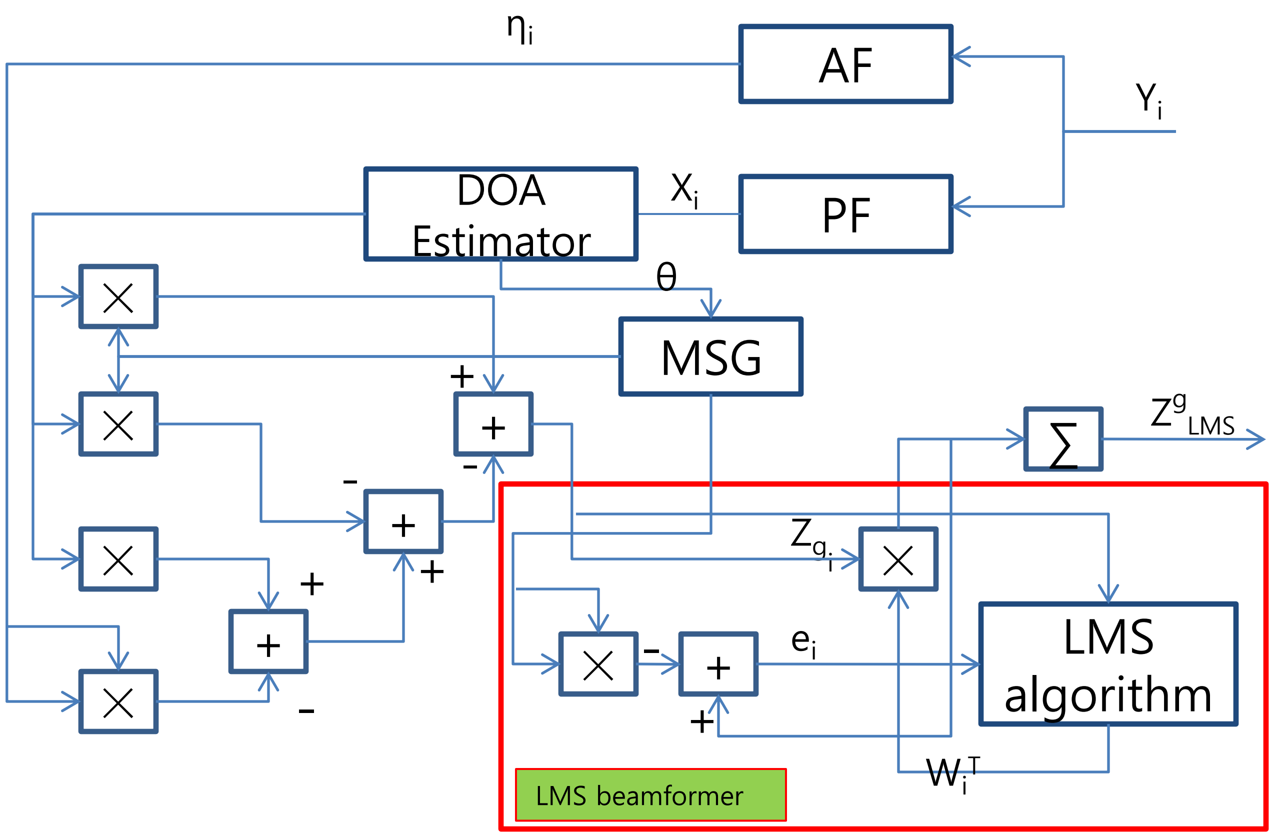


LMS GR and DOA estimation

LMS is a non-blind beamfomring algorithm requiring a priori information about the transmitted signal. In practice, the signal direction is usually an unknown parameter. If the signal direction is the unique unknown parameter, the DOA estimation technique can be applied in LMS GR without changing the beamforming algorithm.
In this proposed approach, we should estimate the signal DOA before the processing the received signal by GR and LMS beamformer. The estimated signal comes in at MSG to generate the reference signal or model signal that differs from the transmitted signal in the input process. This difference is very small owing to high resolution of DOA estimation.

LMS GR and DOA estimation

LMS GR structure with DOA estimation:



Simulation of LMS GR with DOA estimation

At simualtion, a 8-element uniform linear array with half wavelength has been applied to receive the signal; the target signal and interference signal are set as Gaussian random sequences with zero mean and SNR varied within the limits of the range defined as

[8 10 12 14 16 18 19 20 22 24 26 28] dB.

The DOAs are set as:

DOA_signal=-60;

DOA_I_1=10;

DOA_I_2=60;

The received signal of array or the IMS GR PF output is given by

Xi=Xs+X1+X2+Xn2;

DOA estimator estimates the DOA of signal evaluating the process Xi.

Simulation of LMS GR with DOA estimation

First, using the eigenvalue decomposition of array covariance matrix to obtain the noise subspace:

```
Rx=x*x';                                    %covariance matrix
[V,D]=eig(Rx);                              %eigenvalue decomposition
noise_subspace=V(:,1:N-P);                  %noise_subspace
```

Then, we obtain:

In the case of SNR=28dB

```
D =

  1.0e+003 *

    0.0009         0         0         0         0         0         0         0
         0    0.0009         0         0         0         0         0         0
         0         0    0.0010         0         0         0         0         0
         0         0         0    0.0010         0         0         0         0
         0         0         0         0    0.0011         0         0         0
         0         0         0         0         0    4.4178         0         0
         0         0         0         0         0         0    4.7879         0
         0         0         0         0         0         0         0    6.0118
```

Simulation of LMS GR with DOA estimation

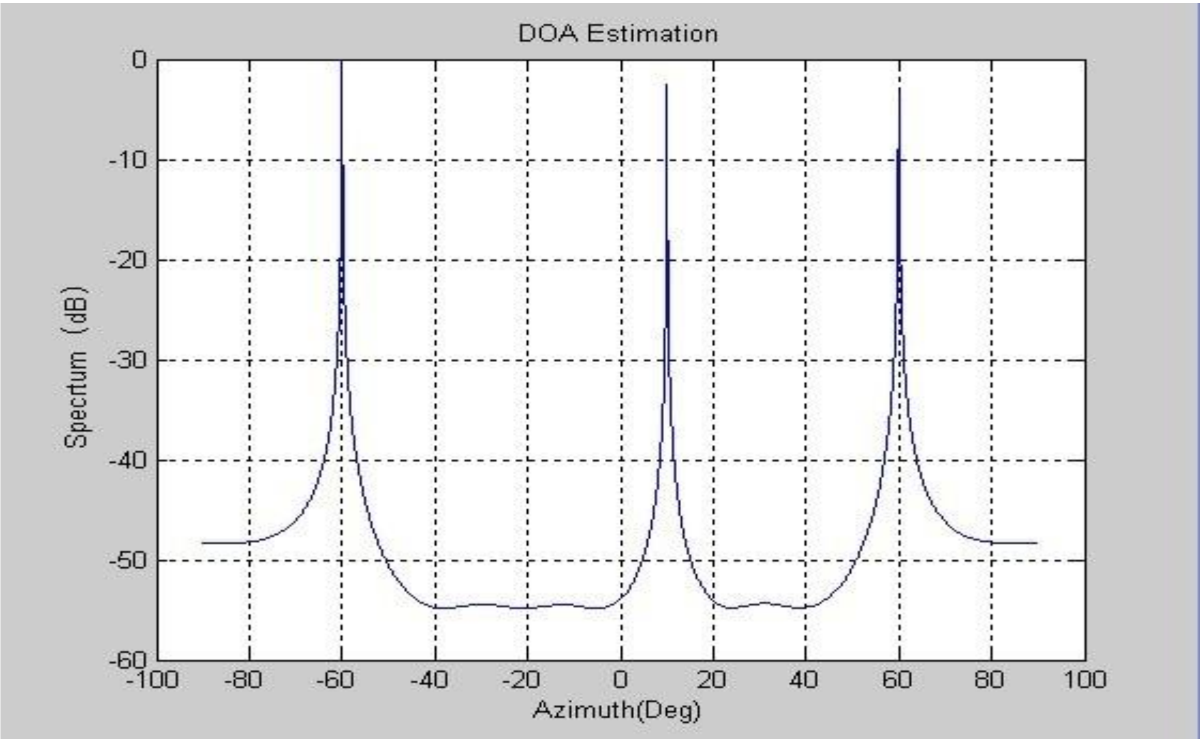
```
V =

  Columns 1 through 4

   0.3966 - 0.2085i  -0.3964 - 0.0248i   0.2707 - 0.2238i  -0.0052 + 0.3349i
   0.5663 + 0.0712i   0.1203 - 0.1278i  -0.3335 - 0.2714i   0.1571 + 0.0591i
   0.0791 - 0.0228i   0.1927 - 0.0603i  -0.3197 + 0.2609i   0.0998 - 0.2888i
  -0.1316 - 0.2113i   0.0028 - 0.0328i   0.2744 + 0.4249i   0.4420 + 0.2140i
  -0.0284 + 0.0442i  -0.3702 + 0.0342i  -0.0262 + 0.0510i   0.3767 + 0.2533i
   0.1457 + 0.0718i  -0.4763 + 0.2279i  -0.2031 - 0.0108i  -0.1576 - 0.3854i
   0.4576 - 0.2377i  -0.0471 + 0.2127i   0.4301 + 0.0697i  -0.0747 - 0.2454i
   0.3312             0.5524             0.1763             0.2828

  Columns 5 through 8

  -0.2254 - 0.0512i  -0.0642 - 0.3204i  -0.1420 + 0.0718i   0.2620 + 0.3894i
   0.1901 + 0.1842i  -0.2961 + 0.1178i   0.0891 + 0.2624i  -0.4232 + 0.0541i
  -0.3182 + 0.4295i  -0.0128 - 0.5570i  -0.0404 - 0.2554i   0.0079 + 0.1604i
  -0.1659 + 0.1681i   0.1357 + 0.0132i  -0.1865 + 0.4477i  -0.3570 - 0.0541i
   0.5155 + 0.0086i   0.0690 - 0.3197i   0.1081 - 0.4416i  -0.1628 - 0.2090i
   0.1841 + 0.2049i   0.4414 - 0.0014i  -0.3053 + 0.3206i   0.0406 - 0.1086i
  -0.2230 + 0.1452i   0.0230 + 0.1978i   0.2187 - 0.3278i  -0.1542 - 0.3761i
   0.3500             0.3481            -0.1944             0.4389
```

Noise Subspace

Signal Subspace

Simulation of LMS GR with DOA estimation

Then, plotting the MUSIC Pseudospectrum and searching the spectr-Um peak, we can estimate the signal DOA.



MUSIC Pseudospectrum

Simulation of LMS GR with DOA estimation

The following results show the final estimation in difference SNR,

XDOA =

-59.9885	9.9926	60.0118
-59.9426	9.9926	60.0233
-60.0114	9.9811	59.9660
-60.0228	10.0155	60.0004
-59.9885	9.9811	60.0118
-59.9999	9.9926	60.0004
-59.9999	9.9811	60.0004
-59.9999	9.9811	60.0004
-59.9885	9.9811	60.0004
-60.0114	9.9811	60.0004
-59.9999	9.9811	60.0004

SNR

8dB

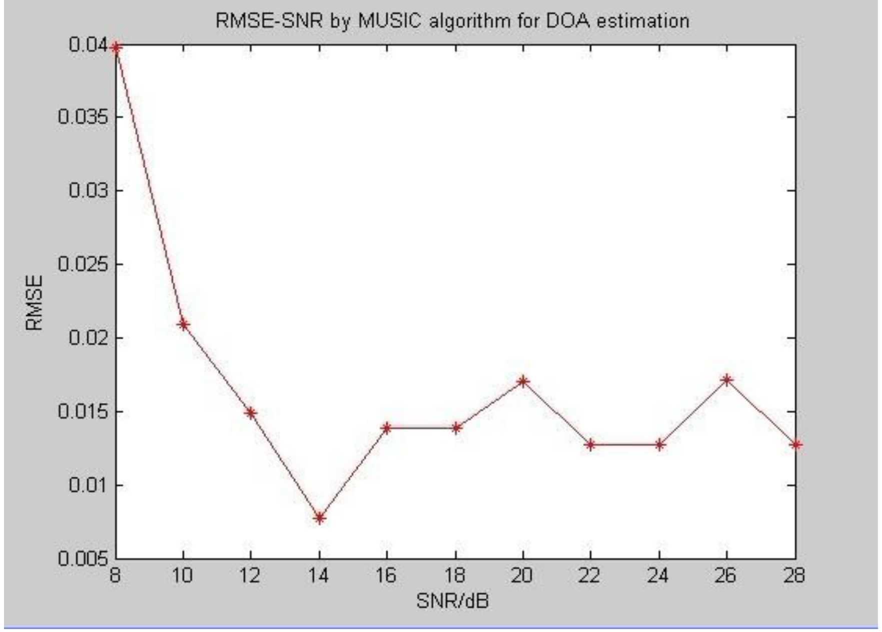
28dB

Simulation of LMS GR with DOA estimation

The root mean square error (RMSE) criterion is employed to assess and compare the DOA estimation results of algorithms in a quantitative manner, and it is determined as

$$RMSE = \sqrt{\sum_{i=1}^{K} \sum_{p=1}^{P} (\theta_p - \hat{\theta}_{i,p})^2 / (KP)}$$

Where, K is the number of Monte-Carlo experiments
P is the number of signals, θ_p is the p th DOA and $\hat{\theta}_{i,p}$ denotes the p th estimated DOA in the i th Monte-Carlo experiment.



RMSE versus SNR of MUSIC algorithms

Simulation of LMS GR with DOA estimation

DOA estimation result comes in at LMS GR MSG input in for interfe-rence cancellation and further signal processing. Comparative analy-sis between the LMS GR with the real angle and with DOA and the Neyman-Pearson receiver with real angle and with DOA demonstra-tes a superiority of the first over the last.

Following equations are the output statistics of LMS NP and LMS GR:

$$Z^{g}_{LMS} = \sum_{i=1}^{N}[W_i^T(a_i^2 - 2I_i\xi_i - I_i^2 + \eta_i^2 - \xi_i^2)].$$

$$Z^{NP}_{LMS} = \sum_{i=1}^{N}[W_i^T(a_i^2 + a_iI_i + a_i\eta_i)].$$

$$a_i^* = a_i$$

Simulation of LMS GR with DOA estimation

$$Z_{LMS}^{g} = \sum_{i=1}^{N}[W_i^T(2a_i^E(a_i + I_i + \xi_i) - (a_i + I_i + \xi_i)^2 + \eta_i^2)].$$

$$\approx \sum_{i=1}^{N}(a_i^E a_i + \eta_i^2 - \xi_i^2)$$

$$Z_{LMS}^{NP} = \sum_{i=1}^{N}[W_i^T(a_i a_i^E + a_i^E I_i + a_i^E \eta_i)].$$

$$\approx \sum_{i=1}^{N}(a_i^E a_i + a_i^E \eta_i)$$

$$a_i^* = a_i^E$$

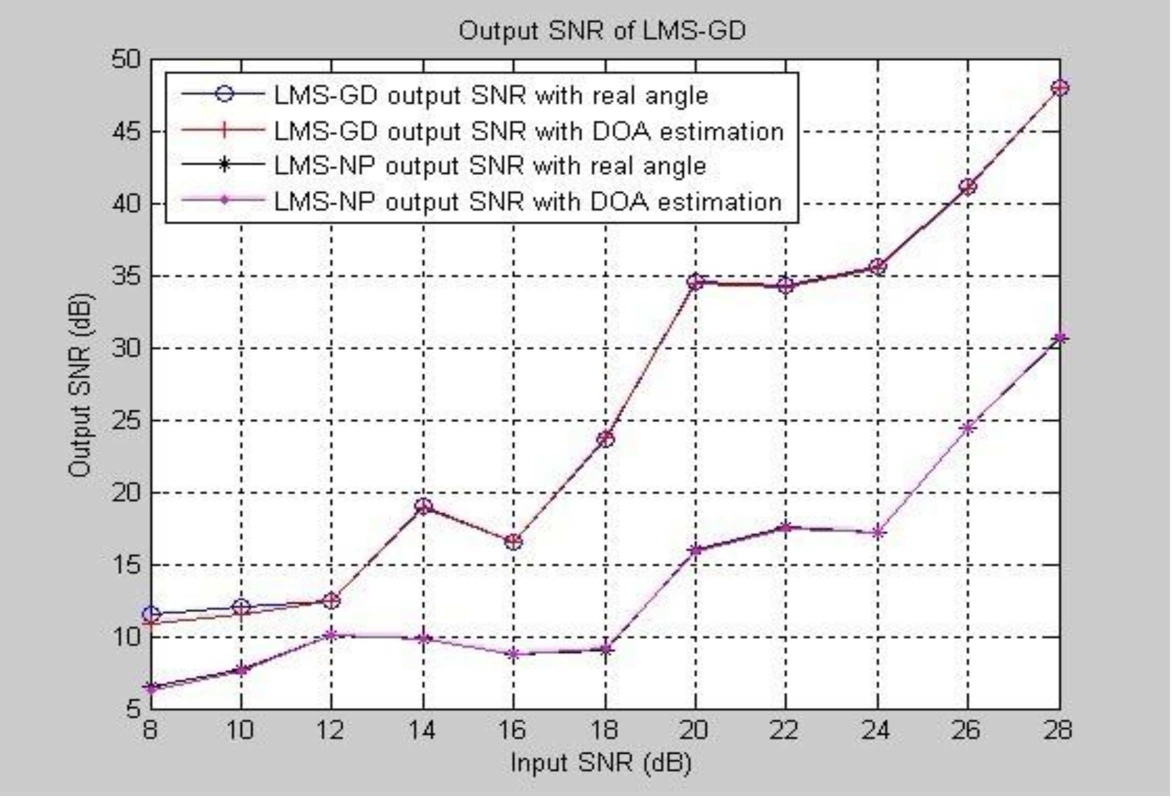
The ratio of signal energy to noise and remaining interference signal at the outputs of LMS NP and LMS GR can be defined as:

$$S\_NI\_GR = \sum_{i=1}^{N} \frac{W_i^T a_i^{E^2}}{Z_{LMS}^{g} - W_i^T a_i^{E^2}}$$

$$S\_NI\_NP = \sum_{i=1}^{N} \frac{W_i^T a_i^{E^2}}{Z_{LMS}^{NP} - W_i^T a_i^{E^2}}$$

Simulation of LMS GR with DOA estimation

Based on analysis, we can compare the performance of LMS GR and NP with the real angle and with the DOA estimation



Conclusion

DOA estimation technique plays an important role in array signal Processing. It is possible to obtain the signal DOA information based on array response. Subspace algorithms are the representative ones in DOA estimation technique, especially, MUSIC algorithm and ESPRIT algorithm that can be applied in any signal processing area. This research deals with the combination of MUSIC DOA estimation algorithm and LMS GR when the DOA is unknown.LMS GR with DOA estimation demonstrates a high performance even if the interference is present and outperforms the NP receiver.

Thank You!!