FFH and MCFH Spread-Spectrum Wireless Sensor Network Systems Based on the Generalized Approach to Signal Processing

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Abstract: - In this paper, the performance of frequency-hopping spread-spectrum wireless sensor network systems based on the generalized approach to signal processing in the presence of noise [1-5], which employ noncoherent reception and transmission diversity, is analyzed for frequency-selective Rayleigh fading channels. Two different types of transmission diversity systems, a fast frequency-hopping (FFH) [6,7] and a multicarrier frequency-hopping (MCFH)[8] wireless sensor network systems, are investigated. In order to combine received signals from transmit diversity channels, the diversity combining rule based on the generalized approach to signal processing is developed. Probability of error equations are derived and utilized to evaluate the performance of two kinds of wireless sensor network systems. The effect of frequency-selective fading is also investigated in determining optimum frequency deviations of binary frequency-shift keying (BFSK) signals. The systems considered in this paper are frequency-hopping spread-spectrum (FHSS) ones with BFSK modulation, noncoherent detection, and definite diversity order. Diversity order refers to the number of hops per symbol for FFH and the number of subbands for MCFH wireless sensor network systems. Each transmit diversity channel is modeled as a frequency-selective Rayleigh fading process and is assumed to be independently faded. The maximum delay spread of each diversity reception is assumed smaller than one hop duration for FFH wireless sensor network systems, which is smaller than the symbol duration. It is also assumed that one symbol is transmitted during one hop duration in MCFI-I wireless sensor network systems, and adjacent symbols in time are transmitted in far distant frequency slot such that multipath interference from the previous symbol is negligible.

Key~ Words: - Wireless sensor network, frequency-hopping spread-spectrum, frequency-shift keying signals, generalized detector, probability distribution function, BER performance.

1 Introduction

Demands for high *data* rate services in wireless sensor networks have been increasing over past few decades. By this reason, there is a sense to use a frequency-hopping spread-spectnun (FHSS) technique in wireless sensor network systems. In high data rate wireless sensor network systems, the effects of frequency-selective fading should be considered due to an increase in the ratio of delay spread to symbol duration. The effects of frequency-selective fading on a FHSS system employing orthogonal BFSK signals are discussed in [9,10] under the assumption that the frequency separation between hvo orthogonal BFSK signals is large enough for the correlation between two detector outputs to be negligible. In practice, it is advantageous to use the minimum frequency separation in multiple-access environments to increase the number of frequency slots for a given total bandwidth [11]. When the minimum frequency separation is employed, the correlation between two detector outputs because of frequency-selective fading and fast fading may be significant.

The use of transmission diversity ensures protection against jamming, multiple-access interference, and fading. For FHSS wireless sensor network systems, the diversity may be realized in the form of fast frequency-hopping (FHH) and multicarrier transmission. FFH is a conventional diversity technique in FHSS wireless sensor network systems. Multicarrier transmission is an alternative diversity technique in FHSS wireless sensor network systems. In FHSS wireless sensor network system, diversity is obtained by changing a transmit frequency more than once over one symbol duration. The transmit frequency is selected from the entire transmit frequency band. In MCFH wireless sensor network system, a total frequency band is partitioned into several disjoint subbands on which replicas of the same signal are transmitted simultaneously. Each replica hops independently in its subband.

In FHSS wireless sensor network systems, coherent demodulation for BFSK signals is relatively difficult. For these systems, the frequency-shift keying (FSK) modulation with noncoherent demodulation is typically employed. In the present paper, BFSK modulation and noncoherent demodulation are assumed to be employed for both FFH and MCFH wireless sensor network systems. Diagrams of FFH and MCFH systems are presented in more detail in [12]. The use of FFH technique in high data rate wireless sensor network system may not be feasible due to its high speed requirements. For wireless sensor network systems employing transmission diversity, diversity receptions should be combined in some way in the receiver. The optimum combining schemes based on the maximum-likelihood criterion have been developed only for static and frequency-nonselective slowly varying channels [6,7,13].

For static channels with partial-band interference, the optimum combining is the sum of the logarithms of zero-order modified Bessel functions [7]. For slow and frequency-nonselective Rayleigh fading channels, the optimum combining rule, given that all of the diversity receptions have the same power spectral density (PSD) of background noise, is square-law equal-gain combining [13]. The optimum combining rule for frequency-selective fast varying and frequency -nonselective slowly varying Rayleigh fading channels with the background noise PSD of each diversity reception not being equal is discussed in [12]. In this paper, we consider the optimum combining rule for frequency-selective fast varying and frequency-nonselective slowly varying Rayleigh fading channels based on the generalized approach to signal processsing in the presence of noise.

2 Channel Model

Consider FHSS wireless sensor network system with BFSK modulation, noncoherent detection, and diversity order L. Diversity order refers to the number of hops per symbol for FFH and the number of sub-

bands for MCFH wireless sensor network systems. Each transmit diversity channel is modeled as a frequency-selective Rayleigh fading process and is assumed to be independently faded. The maximum delay spread of each diversity reception is assumed smaller than one hop duration for FFH wireless sensor network system, which is smaller than the symbol duration.

We also assume that one symbol is transmitted during one hop duration in MCFH wireless sensor network system, and adjacent symbols in time are transmitted in far distant frequency slots such that multipath interference from the previous symbol is negligible. The baseband equivalent of the signal transmitted from sensor node to sink for FFH and MCFH wireless sensor network systems can be represented in the following form

$$a(t) = \sum_{k=0}^{\infty} \sum_{\ell=0}^{L-1} \sqrt{2E_a} \cos[(\omega_{\ell,k} + b_k \omega_d)t + \varphi_{\ell,k}] \\ \times p_{T_\omega} (t - kT - \ell T_\omega); \text{ FFH system}$$
(1)
$$a(t) = \sum_{k=0}^{\infty} \sum_{\ell=0}^{L-1} \sqrt{2E_a} \cos[(\omega_{\ell,k} + b_k \omega_d)t + \varphi_{\ell,k}] \\ \times p_{T_\omega} (t - kT_\omega), \text{ MCFH system}$$
(2)

where E_a is the transmit energy of each diversity transmission, T is the symbol duration, and T_{ω} is the hop duration; $\omega_{\ell,k}$ and $\varphi_{\ell,k}$ are, respectively, the hop frequency and random phase for the ℓ -th diversity transmission of the *k*-th symbol; $b_k \in \{-1,+1\}$ is the *k*-th data symbol, and $p_T = 1$ for $t \in [0,T]$ and zero, otherwise.

The frequency deviation of a BFSK signal is denoted by $\omega_d = \frac{\omega}{2\pi T_{\omega}} = \frac{\Delta \omega}{2}$, where ω is the normalized frequency deviation and $\Delta \omega$ is the frequency separation between two BFSK signals. When the total transmit energy of a(t) is E_a^{total} , the value of E_a in (1) is E_a^{total} and that of E_a in (2) is $E_a^{total} L^{-1}$. Similarly, the value of T_{ω} in (1) is TL^{-1} and that of T_{ω} in (2) is *T*. Correspondingly, the values of ω_d and $\Delta \omega$ would be different for FFH and MCFH wireless sensor network systems. The channel model is considered as a wide-sense stationary uncorrelated scattering model discussed in [14,15]. The low-pass equivalent impulse response of the ℓ -th diversity channel may be represented as

$$c_{\ell}(t;\tau) = A_{\ell}(t;\tau) \cos[\varsigma_{\ell}(t;\tau)],$$

$$\ell = 0,1,...,L-1 , \qquad (3)$$

where $A_{\ell}(t;\tau)$'s are independent and identically distributed Rayleigh random processes and $\zeta_{\ell}(t;\tau)$'s are independent and identically distributed uniform random processes within the limits of the interval $[0,2\pi]$.

The autocorrelation function of the wide-sense stationary uncorrelated scattering channel is given as [14]

$$R_{c}(\Delta t; \tau, \tau') = 0.5M[c^{*}(t; \tau)c(t + \Delta t; \tau')]$$
$$= R_{c}(\Delta t; \tau)\delta(\tau - \tau'), \qquad (4)$$

where M[...] is the mean and * denotes a complex conjugate operation. Since the channel response for each diversity transmission is assumed independent and identically distributed, the autocorrelation of each channel is the same for all ℓ , by this reason the subscript ℓ is dropped in (4). If we let $\Delta t = 0$ in $R_c(\Delta t; \tau)$, the resulting autocorrelation function $R_c(0; \tau)$ is a multipath intensity profile, and denoted as $I_c(\tau)$. Assuming that the multipath intensity profile is time invariant, the autocorrelation function $R_c(\Delta t; \tau)$ may be represented in the following form

$$R_{c}(\Delta t;\tau) = I_{c}(\tau)\psi_{c}(\Delta t), \qquad (5)$$

where $\psi_c(\Delta t)$ is the autocorrelation function in the Δt variable normalized by $I_c(\tau)$ for all τ [14].

3 Performance Analysis

FFH and MCFH system block diagrams were presented in [Figs. 1 and 2, 12]. After down converting and dehopping, the baseband equivalent of the received signal over the first symbol duration may be presented as

$$\begin{aligned} x(t) &= \sum_{\ell=0}^{L-1} \int_{0}^{T_{\max}} \sqrt{2E_a} A_\ell(t;\tau) \cos[b_0 \omega_d t + \theta_\ell(t;\tau)] d\tau \\ &\times p_{T_\omega} \left(t - \ell T_\omega \right) + n_\ell(t) \,, \quad t \in [0,T] \end{aligned}$$
(6)

for FFH wireless sensor network system and

$$\begin{aligned} x(t) &= \sum_{\ell=0}^{L-1} \int_{0}^{T_{\text{max}}} \sqrt{2E_a} A_\ell(t;\tau) \cos[b_0 \omega_d t + \theta_\ell(t;\tau)] d\tau \\ &+ n_\ell(t), \quad t \in [0,T] \end{aligned}$$
(7)

for MCFH wireless sensor network system. Here $\theta_{\ell}(t;\tau) = \varsigma_{\ell}(t;\tau) + \varphi_{\ell,0},$ (8)

and T_{max} is the maximum delay spread of each diversity channel. The noise $n_{\ell}(t)$ is represented as a

low-pass equivalent additive Gaussian noise process with PSD N_{ℓ} .

We assume that data symbol b_0 is either +1 or -1 with equal probability. Without loss of generality, it is assumed that data symbol b_0 is +1 hereafter. Each diversity reception is demodulated by a noncoherent generalized detector [3].



Figure 1. The noncoherent generalized detector.

The noncoherent generalized detector consists of two branches followed by an envelope detector, as shown in Fig.1. We assume that the generalized receiver is time synchronous to the first arriving signal, i.e., $\tau = 0$. Recall the main functioning principles of the generalized detector [3]. The received signal must pass through preliminary filter (PF). The effective frequency bandwidth of the PF coincides by value with that of the transmitted signal. Thus, the signal with data symbol $b_0 = +1$ passes through the PF_{+1} only, and the signal with data symbol $b_0 = -1$ passes through the PF_{-1} only. The additional filter (AF) is formed in a parallel way to the PF_{+1} and PF_{-1} with the purpose to form a reference sample with a priori information a no signal for generation of jointly sufficient statistics of the mean and variance of the likelihood function.

For simplicity of analysis, we assume that the amplitude-frequency response of the AF is analogous to the amplitude-frequency responses of the PF_{+1} and PF_{-1} over the whole range of parameters, but it is detuned in the resonant frequency relative to the PF_{+1} and PF_{-1} for the purpose of providing uncorrelated processes at the outputs of the PF_{+1} or PF_{-1} and AF. The detuning value must be more than the effective frequency bandwidth of the transmitted signals with data symbol $b_0 = +1$ or $b_0 = -1$ so that the processes at the outputs of the PF_{+1} or PF_{-1} and AF will be uncorrelated.

In static environments, when the data symbol b_0 =+1 is transmitted, the processes at outputs of the PF_{+1} and PF_{-1} can be thought as uncorrelated. However, in fading environments, this is not true, since multipath signal components and signal variation over one hop duration may destruct orthogonality of BFSK signals. By this reason, in a general case, we may think that the processes at outputs of the PF_{+1} and PF_{-1} are correlated. Taking into account the main functioning principles of the generalized detector discussed in more detail in [3], the two generalized detector channel outputs of the ℓ -th diversity reception are denoted, respectively, by $Z_{\ell,1}$ and $Z_{\ell,-1}$, and may be expressed in the following complex form

$$Z_{\ell,1} = \frac{2E_a}{T_\omega} \int_0^{T_{\max}} \int_{\tau}^{T_\omega} A_\ell(t;\tau) e^{j\theta_\ell(t;\tau)} dt d\tau + \frac{1}{T_\omega} \int_0^{T_\omega} [\eta_\ell^2(t) - \xi_\ell^2(t)] dt; \qquad (9)$$
$$Z_{\ell,-1} = -\frac{2\alpha E_a}{T_\omega} \int_0^{T_{\max}} \int_{\tau}^{T_\omega} A_c(t;\tau) e^{j[\Delta\omega_d t + \theta_\ell(t;\tau)]} dt d\tau - \frac{\alpha\sqrt{2E_a}}{T_\omega} \int_0^{T_{\max}} \int_{\tau}^{T_\omega} A_\ell(t;\tau) e^{j[\omega_d t + \theta_\ell(t;\tau)]} \zeta_\ell(t) dt d\tau + \frac{1}{T_\omega} \int_0^{T_\omega} [\eta_\ell^2(t) - \zeta_\ell^2(t)] dt, \qquad (10)$$

where $0 \le \alpha \le 0.5$ is the coefficient characterizing the part of energy of the transmitted signal with the symbol $b_0 = +1$ at the output of the PF_{-1} owing to destruction of BFSK signals orthogonality; $\eta_{\ell}(t)$ is the zero-mean Gaussian noise with the variance σ_n^2 forming at the output of the AF; $\xi_{\ell}(t)$ is the zeromean Gaussian noise with the variance σ_n^2 forming at the output of the PF_{+1} ; $\zeta_{\ell}(t)$ is the zero-mean Gaussian noise with the variance σ_n^2 forming at the variance σ_n^2 forming at the output of the PF_{-1} ; the reference (model) signal $\sqrt{2E_a}e^{-j\omega_d t}$ is formed at the output of MSG_{+1} and the reference (model) signal $\sqrt{2E_a}e^{j\omega_d t}$ is formed at the output of MSG_{-1} , but for the considered case $b_0 = +1$ the last reference signal is equal to zero due to the main functioning principles of the noncoherent generalized detector [2,3].

The effect of destruction of BFSK signals orthogonallity is represented as the first and second terms of (10), which will be referred to, hereafter, as interference component. The second term in (9) and the third term in (10) represent the background noise forming at the output of the generalized detector. The first term in (9) and the first and second terms in (10) are zero-mean complex Gaussian random variables. The last term in (9) and (10) is zero-mean but not Gaussian random variable. The distribution law of the last term in (9) and (10) (the background noise of the generalized detector) is discussed in more detail in [3,4]. The variances of $Z_{\ell,1}$ and $Z_{\ell,-1}$ are given by

$$\begin{aligned} \sigma_{\ell,1}^{2} &= 0.5M[|Z_{\ell,1}|^{2}] \\ &= \frac{2E_{a}^{2}}{T_{\omega}} \int_{0}^{T_{\max}} \int_{0}^{T_{\omega}-\tau} R_{c}(t;\tau) \left(1 - \frac{t+\tau}{T_{\omega}}\right) dt d\tau + \frac{2\sigma_{n}^{4}}{T_{\omega}}; (11) \\ \sigma_{\ell,-1}^{2} &= 0.5M[|Z_{\ell,-1}|^{2}] \\ &= \frac{2\alpha^{2}E_{a}^{2}}{T_{\omega}} \int_{0}^{T_{\max}} \int_{\tau}^{T_{\omega}} R_{c}(t;\tau) \cos(\Delta\omega t) \left(1 - \frac{t+\tau}{T_{\omega}}\right) dt d\tau \\ &+ \frac{\alpha^{2}E_{a}\sigma_{n}^{2}}{T_{\omega}} \int_{0}^{T_{\max}} \int_{\tau}^{T_{\omega}} R_{c}(t;\tau) \cos(\Delta\omega t) \left(1 - \frac{t+\tau}{T_{\omega}}\right) dt d\tau \\ &+ \frac{2\sigma_{n}^{4}}{T_{\omega}} . \end{aligned}$$

The correlation coefficient is given by

$$\rho_{\ell} = \frac{0.5M[Z_{\ell,1}Z_{\ell,-1}]}{\sigma_{\ell,1}\sigma_{\ell,-1}}$$
$$= \left[-\frac{2\alpha E_a^2}{T_{\omega}^2} \int_{0}^{T_{max}} \int_{\tau}^{T_{\omega}T_{\omega}} R_c(t_1 - t_2) e^{j\Delta\omega t_1} dt_1 dt_2 d\tau + \frac{2\sigma_n^4}{T_{\omega}^2} \right] \times \frac{1}{\sigma_{\ell,1}\sigma_{\ell,-1}} .$$
(13)

The decision-making rules are made based on L pairs of noncoherent generalized detector outputs, $R_{\ell,1} = |Z_{\ell,1}|$ and $R_{\ell,-1} = |Z_{\ell,-1}|$ for $\ell \in \{0,1,...,L-1\}$. They should be combined in some way to form decision statistics for the noncoherent generalized detector. To find the optimum diversity combining rule

based on the maximum-likelihood criterion, we should find the conditional joint probability density function (pdf) of noncoherent generalized detector outputs, $R_{\ell,1}$ and $R_{\ell,-1}$ for $\ell \in \{0,1,...,L-1\}$ conditioned on a transmitted data symbol. This pdf is referred to as a likelihood function. Because each diversity reception is assumed independent of each other, the likelihood function for data symbol $b_0 = +1$ can be expressed as

$$F_{R}(r_{0,1}, r_{1,1}, ..., r_{L-1,1}, r_{0,-1}, r_{1,-1}, ..., r_{L-1,-1} | b_{0} = +1)$$

= $\prod_{\ell=0}^{L-1} F_{R_{\ell}}(r_{\ell,1}, r_{\ell,-1} | b_{0} = +1),$ (14)

where $F_{R_{\ell}}(r_{\ell,1}, r_{\ell,-1} | b_0 = +1)$ is the conditional joint pdf of the non-coherent generalized detector outputs for the ℓ -th diversity reception.

Using procedure discussed in [12], finally we can obtain the conditional pdf of $R_{\ell,1}$ and $R_{\ell,-1}$

$$F_{R_{\ell}}(r_{\ell,1}, r_{\ell,-1} | b_0 = +1) = \frac{r_{\ell,1}r_{\ell,-1}}{\sigma_{\ell,1}^2 \sigma_{\ell,-1}^2 (1-|\rho_{\ell}|^2)} \times K_0 \Big(\frac{|\rho_{\ell}| r_{\ell,1}r_{\ell,-1}}{\sigma_{\ell,1} \sigma_{\ell,-1} (1-|\rho_{\ell}|^2)} \Big) \cdot e^{\frac{-\sigma_{\ell,-1}^2 r_{\ell,1}^2 + \sigma_{\ell,1}^2 r_{\ell,-1}^2}{2\sigma_{\ell,1}^2 \sigma_{\ell,-1}^2 (1-|\rho_{\ell}|^2)}},$$
(15)

where $K_0(x)$ is the zero-order modified Bessel function of the second kind. Similarly, the likelihood function for data symbol $b_0 = -1$ will be obtained from (14) and (15), by exchanging $r_{\ell,1}$, $r_{\ell,-1}$, $\sigma_{\ell,1}$, and $\sigma_{\ell,-1}$ in (15), respectively. After straightforward algebraic manipulation and extraction of common terms in the log-likelihood functions, the optimum decision rule is derived as

$$\sum_{\ell=0}^{L-1} \frac{\sigma_{\ell,1}^2 - \sigma_{\ell,-1}^2}{\sigma_{\ell,1}^2 \sigma_{\ell,-1}^2 (1 - |\rho_{\ell}|^2)} \cdot (R_{\ell,1}^2 - R_{\ell,-1}^2) \underset{\hat{b}_0 = -1}{\overset{\hat{b}_0 = +1}{\geq}} 0.$$
(16)

Reference to (16) shows that the decision variable associated with $\hat{b}_0 = +1$ is constructed as the weighted sum of squares of $R_{\ell,1}$ for all ℓ ; the decision variable associated with $\hat{b}_0 = -1$ is constructed in a similar manner. These variables are compared to estimate a transmit symbol. Note that the combining rule is differed from the combining rule in [7], which is developed for static channels. In (16), it can be shown that the ℓ -th weighting factor depends on the variances and correlation coefficient of noncoherent for the generalized detector outputs for the ℓ -th diversity reception. The variance $\sigma_{\ell,1}^2$ is composed of signal and background noise components, and the variance $\sigma_{\ell,-1}^2$ is composed of interference and background noise components.

The numerator $\sigma_{\ell,l}^2 - \sigma_{\ell,-l}^2$ represents a difference between signal power and interference power, since the background noise power in $\sigma_{\ell,1}^2$ and $\sigma_{\ell,-1}^2$ is the same. $\sigma_{\ell, \mathrm{l}}^2 - \sigma_{\ell, -\mathrm{l}}^2$ is the same for all ℓ , when the transmit power is the same and fading process is independent and identically distributed for each diversity channel. The denominator $\sigma_{\ell,l}\sigma_{\ell,-1}(1-|\rho_{\ell}|^2)$ represents that the weighting factor should be small when the noise power increases, $\sigma_{\ell,1}^2$ and $\sigma_{\ell,-1}^2$ increase and $|\rho_{\ell}|$ decreases. To compare the performance of FFH and MCFH wireless sensor network systems and to evaluate the effects of diversity order in typical frequency-selective fading channels, the PSD N_{ℓ} of the additive Gaussian noise for each diversity reception is assumed to be the same, i.e., $N_{\ell} = N_0$, where N_0 is the one-sided thermal noise PSD. From this assumption, the variances and correlation coefficient of the noncoherent generalized detector outputs given by (11)–(13), are the same for all ℓ : $\sigma_{\ell,1}^2 =$ $\sigma_1^2, \ \sigma_{\ell,-1}^2 = \sigma_{-1}^2, \text{ and } \rho_\ell = \rho \text{ for } \ell \in \{0,1,..., L-1\}.$

With this assumption, the optimum combining rule in (16) becomes square-law equal-gain combining, which is the same result as in [13], where orthogonality between BFSK signals is maintained. Based on (16) and the above assumption, the probability of error for the optimally combined signal may be expressed as

$$P_e = \int_{-\infty}^{0} F(D \mid b_0 = +1) \, dD \quad , \tag{17}$$

where *D* is the decision variable defined as $D = \sum_{\ell=0}^{L-1} D_{\ell}$, and $D_{\ell} = R_{\ell,1}^2 - R_{\ell,-1}^2$. $F(D | b_0 = +1)$ is the conditional pdf of *D*, given $b_0 = +1$. The conditional pdf $F(D | b_0 = +1)$ may be found using (14) and (15) with appropriate transformations of random variables. It can be shown that the decision variable *D* in (17) may be viewed as a special case of the general quadratic form studied in [15], where the characteristic function-based approach is presented to obtain a simple closed-form expression for P_e .

Using technique discussed in [12, 15], the probability of error can be obtained as

$$P_{e} = \frac{\gamma^{\ell}}{2\pi j (1+\gamma)^{2L-1}} \sum_{\ell=0}^{L-1} \left(2L_{\ell} - 1\right) \cdot \int_{\Gamma} \frac{d\nu}{\nu^{L-\ell} (1-\nu)} , \qquad (18)$$

where Γ is a circular contour of radius less than unity that encloses the origin, and γ is defined as

$$\gamma = \frac{\sigma_{1}^{2} - \sigma_{-1}^{2} + \sqrt{(\sigma_{1}^{2} + \sigma_{-1}^{2})^{2} - 4 |\rho_{\ell}|^{2} \sigma_{1}^{2} \sigma_{-1}^{2}}}{\sigma_{-1}^{2} - \sigma_{1}^{2} + \sqrt{(\sigma_{1}^{2} + \sigma_{-1}^{2})^{2} - 4 |\rho_{\ell}|^{2} \sigma_{1}^{2} \sigma_{-1}^{2}}}$$
(19)

For $\ell \ge L$, the contour integral is zero by Cauchy's theorem [16], since the integrand in (18) is an analytic function in Γ . However, for $0 \le \ell \le L - 1$, the contour integral should be calculated using Residue theorem [16]. Thus, the probability of error in (22) can be determined by

$$P_e = \sum_{\ell=0}^{L-1} \left(2L - 1 \right) \frac{\gamma^{\ell}}{\left(1 + \gamma \right)^{2L-1}} , \qquad (20)$$

which may be expressed in an alternative form

$$P_{e} = \sum_{\ell=0}^{L-1} \left(L + \ell - 1 \right) \frac{\gamma^{\ell}}{\left(1 + \gamma \right)^{L+\ell}} .$$
 (21)

It is not difficult to prove an equivalence of (20) and (21). It should be noted that when $\rho = 0$, (21) becomes the probability of error equation developed for frequency-nonselective slow Rayleigh fading channel [13].

4 Performance Evaluation

The BER performance of FFH and MCFH wireless sensor network systems is evaluated using (19) and (20). The variances and correlation coefficient in (11)–(13) are required for (20), and calculated by Monte Carlo integration technique [17]. The autocorrelation function of a fading channel in (5) is assumed to be described by an exponential multipath intensity profile and Jakes' fading model [18]

$$R_{c} = (\Delta t; \tau) = \frac{\frac{\mu}{T_{\max}} (e^{-\frac{\mu\tau}{T_{\max}}} - e^{-\mu})}{1 - (1 + \mu)e^{-\mu}} \cdot I_{0}(\omega_{D}\Delta t),$$
(22)

where μ is a decaying factor and set to 0.5; ω_D is the maximum Doppler spread, and $I_0(x)$ is the zeroorder Bessel function of the first kind. Orthogonal signaling, i.e., $\omega = 0$, is implied, unless explicitly specified.

Figures 2 and 3 show the performance of FFH and MCFH wireless sensor network systems based on the generalized approach to signal processing in the pre-



Figure 2. BER performance of FFH wireless sensor Network system for various delay spreads $(L=3, \omega_D T = 0.063).$



Figure 3. BER performance of MCFH wireless sensor network system for various delay spreads (L = 3, $\omega_D T = 0.063$).

sence of noise for several values of maximum delay spreads, when the normalized maximum Doppler spread $\omega_D T = 0.063$. Diversity order is set to 3. The performance of FHSS wireless sensor network systems is found to be significantly degraded in frequency-selective fading environments with delay spread. The performance degradation due to delay spread is found much more severe in FFH wireless sensor network system than in MCFH wireless sensor network systems. This fact can be explained in the following manner. The probability of error may be proved to be monotonically decreasing function of γ by differenttiating (19) with respect to γ . From (11)–(13), and (19), γ is observed to be related to the ratio of T_{max} to T_{ω} , which is defined as an effective delay spread. It can be shown that γ decreases with the effective delay spread, due to an increase in σ_{-1}^2 and a decrease in σ_1^2 and $|\rho|$.

Thus, the value of γ is smaller for FFH than for MCFH wireless sensor network system, for given delay spread, since the effective delay spread for FFH wireless sensor network system is L times larger than that of MCFH wireless sensor network system. In addition, a comparison between FFH and MCFH wireless sensor network systems based on the generalized and Neyman-Pearson receivers is shown in Figs. 2 and 3. The high superiority of the generalized receiver over the Neyman-Pearson detector is obvious. To investigate the effects of correlation between two generalized detector outputs, the performance of FFH and MCFH wireless sensor network systems with the correlation ignored and $T_{\text{max}} = 0.15T$ are obtained by setting $\rho = 0$ in (19) and plotted in Figs. 2 and 3. The large differences between the correlation ignored and not-ignored cases indicate that the correlation should not be ignored.

5 Conclusions

The BER performance of FFH and MCFH wireless sensor network systems based on the generalized approach to signal processing in the presence of noise in frequency-selective Rayleigh fading channels is presented and compared with the BER performance of the same systems under the use of the Neyman-Pearson receiver. The optimum diversity combining rule based on the maximum-likelihood criterion is developed. It is found that the optimum combining is the weighted sum of the squares of non-coherent generalized detector outputs. A weighting factor is shown to depend on the variances and correlation coefficient of noncoherent generalized detector outputs for each diversity reception. Based on the developed optimum diversity combining decision-making rule under the use of the generalized approach to signal processing in the presence of noise, the expressions for the probability of error are derived and evaluated for various channel conditions. It is found that the use of the generalized detector allows us to reach better BER performance.

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