Adaptive Detection of Range-Spread Targets

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Abstract

In this paper, we address an adaptive detection of range-spread targets or targets embedded in Gaussian noise with unknown covariance matrix by the generalized detector (GD) based on the generalized approach to signal processsing (GASP) in noise. We assume that cells or secondary data that are free of signal components are available. Those secondary data are supposed to process either the same covariance matrix or the same structure of the covariance matrix of the cells under test. In this context, under designing GD we use a two-step procedure. The criteria lead to receivers ensuring the constant false alarm rate (CFAR) property with respect to unknown quantities. A thorough performance assessment of the proposed detection strategies high-lights that the two-step design procedure of decision-mak-ing rule in accordance with GASP is to be preferred with respect to the plain one. In fact, the proposed design procedure leads to GD that achieves significant improvement in detection performance under several situation of practical interest. The design of adaptive detection algorithms based on GASP in the case of mismatch is a problem of primary concern for radar applications. We demonstrate two-step design procedure based on GASP ensures minimal loss.

Ⅰ. Introduction

High-resolution radar (HRR) can resolve a target into a number of scattering centers, depending on the range extent of the target and the range resolution capabilities of the radar. In fact, measurements indicate that the radar properties of several targets, such as aircraft, boats, etc. are well modelled as being due primary to reflection from a few isolated points. These specular reflections match quite well with physical features on the target [1]-[3]. The possible improvement depends upon two factors, namely, increasing the range resolution of the radar reduces the amount of energy per cell back-scattered by distributed clutter and resolved scatterers introduce less fluctuation than an unresolved point target. However, this performance improvement is traded for a significant increase of the computational complexity. In the present paper, we deal with the problem of detecting an extended target or targets with unknown amplitudes embedded in Gaussian noise with unknown covariance ma-trix across a number of adjacent range cells which are also referred to in the following as a primary data. For estimation purposes, we resort to a set of secondary data. We will consider the case wherein the power value of primary and secondary data vectors is not the same or more precisely, both groups of data

separately satisfy the homogeneity condition, but the two covariance matrices coincide only up to a scaling factor. This scenario is refereed as a partially homogeneous environment [4]. The design of the adaptive detection algorithms in the case of mismatch is a problem of primary concern for radar applications. Although most of the spacetime adaptive processing detection schemes have been designed employing the assumption that interference returns were the independent and identically distributed (i.i.d.) Gaussian vectors, experimental campaigns have demonstrated that such an assumption is not always verified. The analysis of several space-time adaptive processing algorithms, mostly conducted assuming homogeneity of the secondary data, has shown that inhomogeneities magnify the loss between the adaptive implementation and optimum conditions [6]-[8]. The spatial correlation of the texture is usually unknown and is thus a viable means to cope with this a priori uncertainty. It relies on modeling, at the design stage, and returns as independent Gaussian vectors with possibly different power values. This procedure has been followed in [9], where non-adaptive detectors for range-spread targets embedded in the compound-Gaussian noise with possibly varying texture from cell to cell have been introduced and assesses. Analysis of clutter recordings collected to emulate airborne radars, have shown that the par-tially homogeneous model well describes clutter for moderately low values of the number of primary and secondary data [10]. We derive the GLRT based on the generalized approach to signal processing (GASP) in noise [11]-[16] for a partially homogeneous environment. This work is motivated by two main considerations. The GLRT-based generalized detector (GD) is very time consuming and, hence, difficult to implement for realtime applications. Additionally, the GLRT GD has no known optimality properties and, for homogeneous environment and point-like targets, simplified test statistics may achieve the higher detection probabilities [17]. In that case, the GLRT GD is not a uniformly most powerful (UMP) invariant one, and actually, a UMP-invariant test does not exist, as shown in [18]. The two-step GLRT-based design procedure has been proposed in [17]. The first step is to derive the GLRT GD for the case if the covariance matrix of primary data M is known. The second step is to insert the sample covariance matrix based on the secondary data in place of the true covariance matrix into the test. A possible alternative has been conceived in [19], namely, the first step is to derive the GLRT for the case that only the structure Σ of the covariance matrix is known. A completely adaptive GD is obtained by plugging the sample covariance matrix, based upon secondary data in place of Σ into the previously derived test statistic.

Ⅱ. Body

We assume that data are collected from N sensors and deal with the problem of detecting the presence of a target across L range cells $\mathbf{z}_l, l = 1,\ldots,L$. We suppose that the possible target is completely contained within those data and neglect range migration. It is assumed that a secondary data set $\mathbf{z}_l, l = L+1,\ldots,L(K+1)$ is

available and that each of such snapshots does not contain any useful target echo and exhibits the same structure of the covariance matrix as the primary data. The detection problem to be solved can be formulated in terms of the binary hypotheses test:

$$\begin{cases} \mathcal{H}_0: & \mathbf{z}_l = \mathbf{w}_l; \qquad l = 1,\ldots,L(K+1) \\ \mathcal{H}_1: & \begin{cases} \mathbf{z}_l = \alpha_l \mathbf{p} + \mathbf{w}_l; & l = 1,\ldots,L \\ \mathbf{z}_l = \mathbf{w}_l, & l = L+1,\ldots,L(K+1) \end{cases} \end{cases} \quad (1)$$

where $\mathbf{p}$ denotes the steering vector, and the α_l, $l = 1,\ldots,L$, are unknown deterministic parameters accounting for both the target and the channel effects. As for the noise vectors, we assume that $\mathbf{w}_l$, $l = 1,\ldots L(K+1)$ are the independent zero mean Gaussian vectors with the covariance matrices given by

$$E[\mathbf{w}_l \mathbf{w}_l^*] = \mathbf{M}, \quad l = 1,\ldots,L(K+1) \quad (2)$$

for the homogeneous environment and

$$\begin{cases} E[\mathbf{w}_l \mathbf{w}_l^*] = \mathbf{M}, & l = 1,\ldots,L \\ E[\mathbf{w}_l \mathbf{w}_l^*] = \delta \mathbf{M} & l = L+1,\ldots,L(K+1) \end{cases} \quad (3)$$

for the partially homogeneous environment with $\delta > 0$, where $E[\cdots]$ denotes the mathematical expectation and $*$ denotes the conjugate transpose. Moreover, we suppose that the noise vectors $\mathbf{w}_l$ possess the circular property usually associated with in-phase and quadrature pairs of a wide-sense stationary process. According to the Neyman - Pearson criterion, the optimum solution to the hypotheses testing problem (1) is the likelihood ratio test based on GASP, but for the case at hand, it cannot be implemented since total ignorance of the parameters $\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_L)$, $\mathbf{M}$ and possibly δ is assumed. We resort to GLRT-based decision sche-mes based on GASP. Strictly speaking, the GLRT-based on GASP is tantamount to replace the unknown parameters with their maximum likelihood estimates under each hypothesis based on entirely of data. Processors that implement the plain GLRT will be referred to in the following as the one-step GLRT GD. Receivers implementing modified GLRT statistics derived following the two-step design procedure will be referred to as the two-step GLRT GD. The decision statistics at the GD output can be presented in the following form [13, Chapter 3]:

$$Z_{GD}^{\text{out}}[n] = \begin{cases} \sum_{l=1}^{L} \eta_l^2[n] - \sum_{l=1}^{L} \xi_l^2[n] + \frac{1}{\delta} \sum_{l=LK}^{L(K+1)} \eta_l^2[n] \Rightarrow \mathbf{T}_0 \Rightarrow \mathcal{H}_0 \\ \sum_{l=1}^{L} \alpha_l^2 p_l^2[n] + \sum_{l=1}^{L} \eta_l^2[n] - \sum_{l=1}^{L} \xi_l^2[n] \\ + \frac{1}{\delta} \sum_{l=LK}^{L(K+1)} \eta_l^2[n] \qquad \Rightarrow \mathbf{T}_1 \Rightarrow \mathcal{H}_1. \end{cases} \quad (4)$$

The probability of detection P_D of GD can be presented in the following form:

$$P_{D_{\eta_1,\ldots,\eta_{L(K+1)}}} = 1 - F_{|\mathbf{U}_1\boldsymbol{\eta}^*, \mathbf{S}_\eta} \left\{ \frac{\mathcal{G}}{\sqrt{2}} \delta, SNR \right\}, \quad (5)$$

where $F_{|\mathbf{U}_1\boldsymbol{\eta}^*, \mathbf{S}_\eta}\{\cdot, SNR\}$ denotes the conditional cumulative distribution function (cdf) of the left-hand size of both tests. In Fig.1, the probability of detection P_D of the GLRT GD, AGD, and ASGD are plotted versus SNR at $N = 8, K = 16$, and several values of L. Note that

the case $L=1$ refers to unresolved targets. We see that increasing in the radar resolution capabilities and suitably exploiting them can produce significant detection gain and the corresponding curves of the AGD and GLRT GD intersect, and, in particular, the AGD outperforms the one-step GLRT GD at high values of the probability of detection P_D. For example, at $L=4$, the AGD outperforms the GLRT GD for all values of the probability of detection P_D of practical interest $(P_D > 0.5)$. The ASGD is poorer than the other two receivers, but the loss is less than 2.5 dB at the probability of detection less than 0.9, i.e., $P_D \le 0.9$. This behaviour is valid if $LK > 2N$.

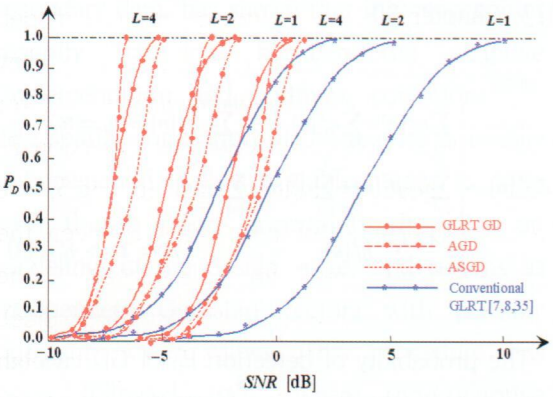

Fig.1. The probability of detection P_D versus SNR of GLRT GD, AGD, and ASGD in homogeneous environment at N=8, K=16, $P_{FA}=10^{-4}$, and L as a parameter.

Finally, the loss of the AGD and ASGD with respect to GLRT GD, namely, the one that possesses perfect knowledge of the covariance matrix M of the noise, can be read off Figs.2,3 at N=8, K as a parameter, $L=2$ and $L=4$. In Fig. 4 we plot the performance of the GLRT GD and the ASGD in partially homogeneous environment at $N=8, K=16$, and several values of L. In this case we do not consider AGD since it is not longer CFAR. We see that the one-step GLRT GD and the ASGD achieve approximately the same performance, but this is not true at $L>1$ as can be shown by simulation for a properly reduced sample size. The ASGD performance and its loss with respect to the GLRT GD can still be read of Figs.1,2 since the ASGD is invariant under scaling of the secondary data. For all cases presented in Figs.1 - 3 we observe a superiority of the GD over the conventional GLRT detector

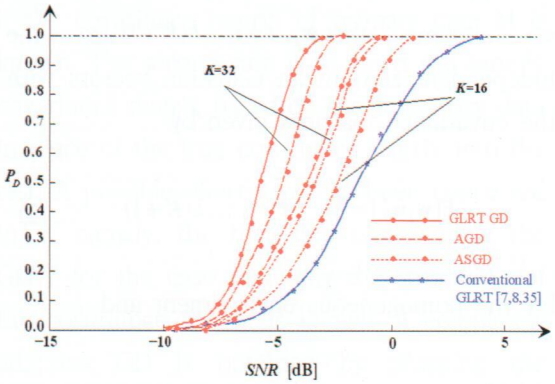

Fig.2. The probability of detection P_D versus SNR of GLRT GD, AGD, and ASGD in homogeneous environment at $N=8$, $L=16$, $P_{FA}=10^{-4}$, and K as a parameter.

We assess the performance of the AGD and ASGD when the unknown deterministic parameters accounting for both the target and the channel $\alpha_l, l=1,\ldots,L$ are the random variables. Obviously, the probability of false alarm P_{FA} is unaffected by the actual characterization of the parameters α_l. Under the hypothesis $\mathcal{H}_1$, the pdf of either statistics is

independent of phase characterization of the parameters parameters $\alpha_l, l=1,\ldots,L$. Thus, should only the phases are random, the probability of detection P_D of GD would not be changed and hence, the curves of Figs.1 - 3 would still be valid. If the amplitudes are random variables, due to the de-pendence of *SNR* on the parameters α_l, different statistical characterizations of the target can result in significantly different probabilities of detection. It is customary to model the $|\alpha_l|^2, l=1,\ldots,L$ as chi-squared random variables. It would be interesting to evaluate the impact on the performance of a degree of correlation among the scattering centers of the target. To this end, we assume that the $|\alpha_l|^2, l=1, \ldots, L$ are drawn from an exponentially correlated random sequence with the one-lag correlation coefficient ρ.

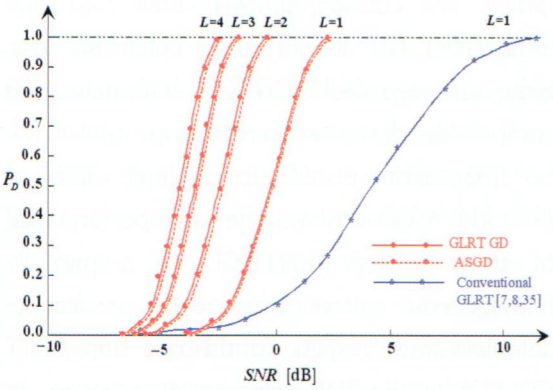

Fig.3. The probability of detection P_D versus *SNR* of GLRT GD, AGD, and ASGD in partially homogeneous environment at $N=8$, $K=16$, $P_{FA}=10^{-4}$, and L as a parameter.

In Fig.4, we analyze an influence of the fluctuation law at $N=8, K=16, L=4, \rho=0$, i.e., the parameters $|\alpha_l|, l=1, \ldots, L$ are independent of each other, the multiple dominant scattering target model 1 from Table 1 [35], and m as a parameter. Any permutation of scatterer positions among the cells under test does not influence the performance, also due to assumption that the parameters $|\alpha_l|, l=1,\ldots,L$ are independent random variables. The AGD performance operating in a homogeneous environment is shown in Fig.5. The performance depends on the actual multiple dominant scattering target model being in force. The probability of detection P_D of GD can be obtained by averaging (5) with respect to the SNR, respectively, and the distribution of the SNR depends on the multiple dominant scattering target model. Figs.1 - 4 show that the fluctuation law significantly affects the performance only for high values of the proba-bility of detection P_D in the medium/ high range. In Fig.5, we analyze the effect of correlation between the target amplitudes for the AGD (the dependences for the ASGD are little bit worse). We refer to $N=8, K=16, L=4$, Rayleigh -fluctuating amplitudes, the multiple dominant scattering target model 1 from Table 1 [35], and several values of the one-lag correlation coefficient ρ. Fig.5 highlights that the correlation between the $|\alpha_l|, l=1,\ldots,L$ is responsible for an additional loss. This behaviour can be easily explained intuitively. In fact, when the received signals from target scatterers are significantly correlated it may happen that all of them "fade at the same time" and this may cause miss-ing of the detection. We note that Figs.1 - 5 highlight

that the GLRT GD, the AGD, and the ASGD outperform the conventional GLRT detector.

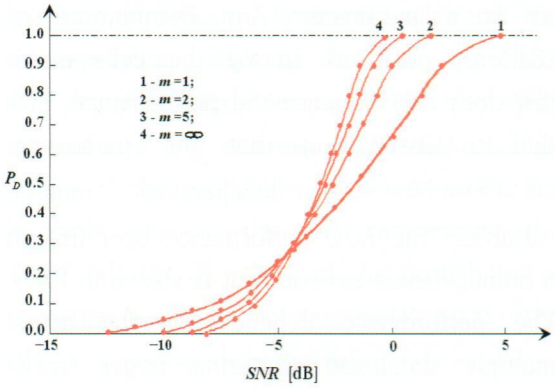

Fig.4. The probability of detection P_D versus *SNR* of AGD in ho-mogeneous environment at N=8, K=16, $P_{FA}=10^{-4}$, and L=4 with chi-fluctuating amplitudes and m as a parameter.

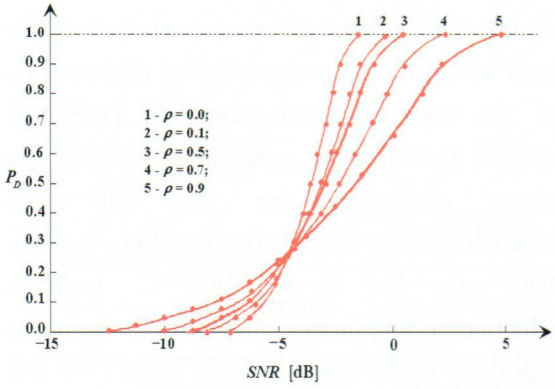

Fig.5. The probability of detection P_D versus *SNR* of AGD in homogeneous environment at N= 8, K= 16, $P_{FA}=10^{-4}$, and L=4 with correlated Rayleigh-distributed amplitudes and ρ as a parameter.

Ⅲ. Conclusion

In this paper, we have addressed the problem of adaptive detection of range-spread targets in homogeneous and partially homogeneous environment. We designed and assessed the one-step and two-step GLRT GD that possesses the CFAR properties. We have shown that the AGD and ASGD have the CFAR property under a homogeneous environment and the one-step GLRT GD and ASGD have the CFAR property under a partially homogeneous environment. As to computational complexity, we have shown that the two-step GLRT GD are faster to implement than the one-step one, and the amount of work required for their implementation grows linearly with the number of range cells L. As to the detection performance, we have derived the analytical dependence of the probability of detection P_D of GD on the target and the noise parameters and estimated the probability of detection P_D of GD through the Monte Carlo simulations. The cases of fluctuating and non-fluctuating targets are considered. We could find that a)the GLRT GD do not suffer collapsing loss; b)the one-step GLRT GD and AGD may have comparable detection performance under the ho- mogeneous disturbance at high values of LK; c)the ASGD achieves the same performance of the one-step GLRT GD in a partially homogeneous environment and has an accept-able loss with respect to the one-step GLRT GD (22) in the homogeneous disturbance. In the latter case, we have focused on the AGD and ASGD and have found that the fluctuation law of the target amplitudes strongly affects the probability of detection P_D of GD in the medium/high range. We have evaluated the impact on the performance of a degree of correlation between the scattering centers of

the target. We have found that the correlation is responsible of an additional loss which is relevant for values of the probability of detection P_D of GD in the medium/high range. In conclusion we state: a)increasing in the radar resolution capa-bilities and suitably exploiting them can produce a significant detection gain; b)the modified GLRT GD is superior to the plain GLRT, as it leads to superior detection performance; c)the ASGD is somewhat more robust than the AGD in that it guarantees CFARness under both scenarios.

We still must assess the capability of the proposed receivers based on GASP in detecting the slightly mismatched signals while rejecting the unwanted signals, i.e. the side-lobe signals. This is a problem of primary concern in a sur-veillance system and is the object of current and future investigations.

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