Performance Analysis under Multiple Antennas in Wireless Sensor Networks Based on the Generalized Approach to Signal Processing

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Abstract: - The average bit-error rate performance of wireless sensor networks based on the generalized approach to signal processing in the presence of noise under the use of multiple antennas at the sensor sink is investigated as a function of the transmit antenna update rate at the sensor nodes when using binary phase-shift keying signals in flat Rayleigh fading channels. This scheme achieves an order of diversity equal to the product of the number of transmit sensor node antennas and receive sensor sink ones. Therefore, it can gain significant diversity benefits over traditional receive diversity schemes by distributing the antennas over transmit and receive side (sensor nodes and sensor sink, respectively). We study two cases: an ideal transmit antenna selection and non-ideal transmit antenna selection.

Key-Words: - Wireless sensor network, antenna diversity, probability of error, generalized detector, Rayleigh fading channel.

1 Introduction

Wireless sensor network systems based on the generalized approach to signal processing in the presence of noise [1]–[5] using multiple antennas at the receiver of sensor sink may draw considerable attention as a means for improving the spectral efficiency or the reliability of wireless sensor network systems over fading channels. The spatial diversity scheme analyzed in this paper forms a combination of selection combining at the transmitter of sensor nodes and maximum-ratio combining (MRC) at the generalized receiver of sensor sink. We assume the transmitter at the sensor nodes is equipped with K antennas and the generalized receiver at the sensor sink possesses L antennas. In the proposed scheme, we assume the transmitter at the sensor nodes selects that antenna out of the K transmit antennas which results in the highest total received power and only transmits data symbol from this antenna.

At the input of the generalized receiver at the sensor sink, the signals originating from the selected transmit antenna of the sensor nodes are coherently combining using the maximum-ratio combining. The average bit-error rate (BER) of this combined spatial diversity scheme is derived when using binary phase-shift keying (BPSK) signals over narrow-band flat Rayleigh fading channels and the performance degradation due to the time-varying nature of the wireless sensor network channels is analyzed.

By using only one transmit antenna at the sensor nodes, computationally efficient one-dimensional processing remains possible. Furthermore, the transmitter at the sensor nodes needs only be equipped with one front-end and an analog switch, substantially reducing the transmitter's complexity and required power. Thus, this scheme can be applied in the uplink of wireless sensor network systems, where transmit diversity utilizing space-time codes [6] or transmit diversity combining [7] may not be applicable due to power and complexity constraints for the sensor sink. Finally, transmit antenna selection at the sensor nodes provides a higher degree of robustness against channel estimation errors and time variation of the channels than transmit diversity schemes which require full channel knowledge at the transmitter of sensor nodes [7].

In this paper, we assume the feedback from the generalized receiver at the sensor sink to the transmitter at the sensor nodes, required for the antenna selection, is neglected since the amount of feedback information is limited compared to full channel knowledge transmit diversity schemes [7]. Furthermore, we obtain in this paper that the required feedback rate for time-varying channels is limited. Hence, the resulting overall overhead is small.

An alternative transmit/receive diversity scheme discussed in [8] utilizes selection diversity both at the transmitter and receiver sides (the sensor nodes and the sensor sink). When only the transmit/receive antenna pair with the highest channel co-efficient is used, this alternative scheme is identical to $K \times L$-fold receive selection combining scheme, which achieves an order of diversity $K \times L$. As was shown in [9], the selection combining/maximum-ratio combining also obtains an order of diversity equal to $K \times L$. However, the additional maximum-ratio combining gain determined by L ensures the better performance in comparison with the alternative combined transmit/diversity scheme [9].

2 System Model

The sampled complex baseband signals received at the l-th antenna of the generalized detector in the i-th signaling interval, when using only transmit antenna k, can be written as

$$x_{kl}[i] = \alpha_{kl}[i]s[i] + n_{kl}[i] \ , \tag{1}$$

where $\alpha_{kl}[i]$ is the time-varying complex channel gain from antenna k to antenna l, $s[i]$ is the transmitted symbol, and $n_{kl}[i]$ is a zero-mean complex Gaussian variable representing the additive white Gaussian noise with variance σ_n^2.

Each channel gain is assumed to be a complex variable with a Rayleigh distributed amplitude and a uniformly distributed phase within the limits of the interval $[-\pi, \pi]$. Furthermore, all gains are assumed to be independent and identically distributed. The autocorrelation function of the coefficients $\alpha_{kl}[i]$, according to Jakes' model, is given by

$$R_{\alpha\alpha}[i] = J_0(2\pi F_D T_s i) \ , \tag{2}$$

where $J_0(\cdot)$ stands for the zero-order Bessel function, F_D is the Doppler fading rate (assumed equal for all channels), and T_s is the symbol period.

For each transmit antenna $k = 1 \cdots K$, the total instantaneous signal-to-noise ratio (SNR) per bit $q_k[i]$ at the output of the generalized detector after performing maximum-ratio combining is given by [2]

$$q_k[i] = \frac{E_b}{\sqrt{4\sigma_n^4}} \sum_{l=1}^{L} \alpha_{kl}[i] \cdot \alpha_{kl}^*[i] \ . \tag{3}$$

Here, $*$ denotes the complex conjugate and E_b is the average energy per bit in the transmitted symbols $s[i]$. $4\sigma_n^4$ is the variance of the process observed at the output of the generalized receiver at the sensor sink. The average bit SNR per channel is defined as

$$\overline{q} = \frac{E_b}{\sqrt{4\sigma_n^4}} M\{\alpha_{kl}[i] \cdot \alpha_{kl}^*[i]\} \ , \tag{4}$$

where $M\{\cdot\}$ is the mean, and does not depend on k, l, or i. At time $i = 0$, the transmit antenna b at the sensor nodes with the largest total SNR $q_b[0]$ is selected.

As was mentioned in [9], the performance of selection combining/maximum-ratio combining scheme depends heavily on the ability to track time-varying channel coefficients. The maximum-ratio combining coefficients can easily be updated by inserting pilot symbols or by using a decision-directed channel estimation approach. Therefore, the perfect knowledge of the maximum-ratio combining coefficients is assumed. Updating the optimal transmit antenna at the sensor node however requires estimation of the channel parameters for each transmit antenna at the sensor nodes and receiver-transmitter feedback. In practical applications, the feedback rate will necessarily be limited, resulting in performance degradation. A trade-off dependent on the Doppler fading rate of the channel can be made between performance loss and overhead.

3 Probability of Error

We compute the average probability of error $\overline{P}_{er}[i]$ for binary phase-shift keying signals at the output of the generalized receiver at the sensor sink using the procedure discussed in [10]. For this purpose, we average the probability of error $\overline{P}_{er}[i]$ conditional on the instantaneous bit SNR $q_b[i]$ over the probability distribution density $f_b(q_b[i])$ of $q_b[i]$

$$\overline{P}_{er}[i] = \int_0^\infty \Phi\{q_b[i]\} f_b(q_b[i]) d(q_b[i]) \ , \tag{5}$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-0.5u^2} du \tag{6}$$

is the error integral.

The average probability of error $\overline{P}_{er}[i]$ is computed for two cases: a) sufficiently high feedback rate resulting in an optimal transmit antenna selection at the sensor nodes and b) the effect of a limited feedback rate is taken into consideration. Consider these two cases.

3.1 Ideal Transmit Antenna Selection

Since the symbol rate should be many times faster than the channel fading rate, the transmit antenna update rate at the sensor nodes can also be made significantly higher than the fading rate of the channels. Then, the channels remain almost constant between updates and the time index i can be dropped. The total instantaneous SNRs per bit q_k are distributed according to a central chi-square probability distribution density $f(q_k)$ with $2L$ degrees of freedom [10]

$$f(q_k)=\frac{q_k^{L-1}}{(L-1)!\overline{q}^L}\exp\Big(-\frac{q_k}{\overline{q}}\Big)\ . \tag{7}$$

The probability distribution function $F(q_k)$ for the aggregate SNRs per bit q_k is given by

$$F(q_k)=1-\exp\Big(-\frac{q_k}{\overline{q}}\Big)\sum_{l=0}^{L-1}\frac{1}{l!}\Big(\frac{q_k}{\overline{q}}\Big)^l\ . \tag{8}$$

Since these q_k are independent, the probability distribution density $f_b(q_b)$ of the resulting instantaneous SNR per bit

$$q_b=\max_{k=1\cdots K}\{q_k\} \tag{9}$$

can be obtained directly in the following form

$$f_b(q_b)=KF(q_b)^{K-1}f(q_b)\ . \tag{10}$$

By expanding (10) with the multi-nominal formula and combining it with (5), the average probability of error $\overline{P}_{er}$ can be written in the following form

$$\overline{P}_{er}=\frac{K!}{(L-1)!}\sum_r\Big[\frac{b_r!I(a_r,b_r)}{a_r^{b_r+1}\prod_{l=0}^{L}n_{r,l}!}\prod_{l=0}^{L-1}\Big(\frac{-1}{l!}\Big)^{n_{r,l}}\Big]\ , \tag{11}$$

where

$$a_r=\sum_{l=0}^{L-1}n_{r,l}+1 \tag{12}$$

and

$$b_r=\sum_{l=0}^{L-1}l\,n_{r,l}+L-1\ . \tag{13}$$

The sum over r in (11)–(13) extends over all possible sets S_r of $L+1$ positive integers $n_{r,l}$ such that

$$\sum_{l=0}^{L}n_{r,l}=K-1\ . \tag{14}$$

The basic integral $I(a_r,b_r)$ used in (11) is given by [10]

$$I(a_r,b_r)=\Big(\frac{a_r}{\overline{q}}\Big)^{b_r+1}\int_0^\infty\frac{x^{b_r}}{b_r!}e^{-\frac{a_r x}{\overline{q}}}\Phi(x)dx$$

$$=\Big(\frac{1-\mu_r}{2}\Big)^{b_r+1}\sum_{i=0}^{b_r}\binom{b_r+i}{i}\Big(\frac{1+\mu_r}{2}\Big)^i\ , \tag{15}$$

where

$$\mu_r=\sqrt{\frac{\overline{q}}{\overline{q}+a_r}}\ . \tag{16}$$

3.2 Nonideal Transmit Antenna Selection

When the update rate is not sufficiently high compared to the Doppler fading rate, the channels fade significantly between updates of the optimal transmit antenna at the sensor nodes and severe performance degradation occurs. When the update rate is too low, the diversity gain from the transmit antenna array at the sensor nodes disappears and only L-fold maximum-ratio combining diversity remains.

In order to compute the average probability of error $\overline{P}_{er}[i]$ we use the procedure proposed in [11]. The probability distribution density $f_b(q_b[i])$ can be determined in the following form

$$f_b(q_b[i])=K\Big\{\int_0^\infty f(q_1[0])\,F(q_1[0])^{K-1}\times f(q_1[i]\,|\,q_1[0])\,dq_1[0]\Big\}\Big|_{q_1[i]=q_b[i]}\ . \tag{17}$$

The conditional probability distribution density $f(q_1[i]\,|\,q_1[0])$ is found using the conditional probability distribution density $f(x[i]\,|\,x[0])$ for a Gaussian variable $x[i]$ with the autocorrelation function $R_{xx}[i]$ [11]

$$f(x[i]\,|\,x[0])=\frac{1}{\sqrt{2\pi}\sigma_i}\exp\Big(-\frac{(x[i]-kx[0])^2}{2\sigma_i^2}\Big), \tag{18}$$

where

$$k=\frac{R_{xx}[i]}{R_{xx}[0]} \tag{19}$$

and

$$\sigma_i^2=R_{xx}[0]-\frac{R_{xx}^2[i]}{R_{xx}[0]}\ . \tag{20}$$

By applying (18)–(20) to each of the $2L$ independent components of $q_1[i]$, the probability distribution density $f(q_1[i]\,|\,q_1[0])$ can be found in the following form [9]

$$f(q_1[i] \,|\, q_1[0]) = \frac{1}{\overline{q}_1}\Big(\frac{q_1[i]}{k^2 q_1[0]}\Big)^{\frac{L-1}{2}}$$
$$\times \exp\Big(-\frac{q_1[i]+k^2 q_1[0]}{\overline{q}_i}\Big)\cdot I_{L-1}\Big(\frac{2k\sqrt{q_1[i]q_1[0]}}{\overline{q}_i}\Big), \tag{21}$$

where $I_{L-1}(\cdot)$ is the modified Bessel function of the first kind of order $L-1$,

$$k = \frac{R_{\alpha\alpha}[i]}{R_{\alpha\alpha}[0]}, \tag{22}$$

and

$$\overline{q}_i = \frac{E_b}{\sqrt{4\sigma_n^4}}\Big(R_{\alpha\alpha}[0] - \frac{R_{\alpha\alpha}^2[i]}{R_{\alpha\alpha}[0]}\Big) \tag{23}$$

with $R_{\alpha\alpha}[i]$ defined in (2). This conditional probability distribution density is a time-dependent, noncentral chi-square distribution with $2L$ degrees of freedom which tends to approach a central chi-square distribution as $i \to \infty$, implying that in the limit only the maximum-ratio combining diversity remains.

After inserting $f(q_1[i] \,|\, q_1[0])$ in (17), using the multinomial formula, and solving the integral by changing the variables from $q_1[0]$ to $r_1 = \sqrt{q_1[0]}$ and using the tabulated integral [12], the final probability distribution density $f_b(q_b[i])$ is given by

$$f_b(q_b[i]) = \frac{K!\, q_b^{L-1}[i]}{[(L-1)!]^2\, \overline{q}_i^L}\exp\Big(-\frac{q_b[i]}{\overline{q}_i}\Big)$$
$$\times \sum_r \Big[\frac{b_r!}{(\overline{q}c_r)^{b_r+1}\prod_{l=0}^{L} n_{r,l}!}\cdot {}_1F_1\Big(b_r+1, L; \frac{k^2 q_b[i]}{\overline{q}_i^2 c_r}\Big)$$
$$\times \prod_{l=0}^{L}\Big(\frac{-1}{l!}\Big)^{n_{r,l}}\Big], \tag{24}$$

where

$$c_r = \frac{a_r}{\overline{q}} + \frac{k^2}{\overline{q}_i} \tag{25}$$

and ${}_1F_1(\cdot)$ stands for the confluent hypergeometric function [10]. The average probability of error at signaling interval i as given by (5), could not be solved in closed form. By this reason, this integral is evaluated numerically.

4 Numerical Results

In [9] the analysis of the average probability of error versus average SNR per channel for different setups with a total number of antennas $K+L=8$ when the feedback rate is sufficiently high is presented in more detail for the maximum-ratio combining. It was shown, for example, that the case $(K,L)=(1,7)$ corresponding to receive maximum-ratio combining outperforms significantly the case $(K,L)=(7,1)$ corresponding to transmit selection combining. Also, it was presented that for the SNR region of interest the optimal distribution of antennas corresponds to the case $(K,L)=(3,5)$ with diversity order 15, due to the higher maximum-ratio combining gain.

The main purpose of numerical results in this paper is to show superiority under an employment of the generalized receiver in wireless sensor network systems over, for example, the use of the Neyman–Pearson detector. Figure 1 shows the average probability of error versus average SNR per channel $\overline{q}$ for the optimal distribution of antennas $(K,L)=(3,5)$ in the maximum-ratio combining case when the feed-back rate is sufficiently high. The results presented in Fig. 1 show us that the generalized receiver in wireless sensor network systems under maximum-ratio combining at the sensor sink significantly outperforms the Neyman–Pearson detector.

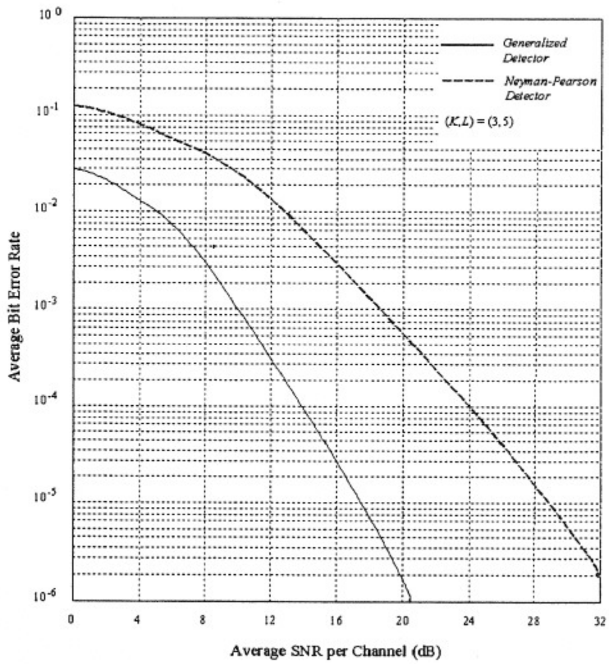

Figure 1. Average bit-error probability versus average SNR $\overline{q}$ at the detector output for binary phase-shift keying signals in independent identically distributed Rayleigh fading.

5 Conclusions

The average probability of error under an employment of the generalized receiver in wireless sensor network systems for combined antenna selection at the sensor nodes with maximum-ratio combining at the sensor sink in flat Rayleigh fading for binary phase-shift keying signals was derived. This simple combined transmit/receive spatial diversity scheme achieves an order of diversity equal to the product of transmit antennas at the sensor nodes and receive antennas at the sensor sink. A superiority of employment of the generalized receiver in wireless sensor network systems over the Neyman–Pearson detector under the use of combined antenna selection at the sensor nodes with maximum-ratio combining at the sensor sink in flat Rayleigh fading for binary phase-shift keying signals is demonstrated.

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References:

[1] V. Tuzlukov, *Signal Processing in Noise: A New Methodology*. Minsk: IEC, 1998.

[2] V. Tuzlukov, "A new approach to signal detection", *Digital Signal Processing: A Review Journal,* Vol. 8, No. 3, pp. 166–184, 1998.

[3] V.Tuzlukov, *Signal Detection Theory.* New York Springer-Verlag, 2001.

[4] V.Tuzlukov, *Signal Processing Noise.* Boca Raton, London, New York, Washington D.C.: CRC Press, 2002.

[5] V. Tuzlukov, *Signal and Image Processing in Navigational Systems.* Boca Raton, London, New York, Washington D.C.: CRC Press, 2004.

[6] V. Tarokh, H. Jafarkani, and A.R. Calderbank, "Space-time block codes from orthogonal designs", *IEEE Trans. Inform. Theory,* Vol. 45, No. 7, 1999, pp. 1456–1467.

[7] A. Wittneben, "Analysis and comparison of optimal predictive transmitter selection and combining diversity for DECT", in *Proc. GLOBECOM,* Montreal, PQ, Canada, June 1995, pp.1527–1531

[8] N.R. Sollenberger, "Diversity and automatic link transfer for a TDMA wireless access link", in *Proc. GLOBECOM,* 1993, pp. 532–536.

[9] S. Thoen, L. Van der Perre, B. Gyselinckx, and M. Engels, "Performance analysis of combined transmit-SC/receive-MRC", *IEEE Trans. on Commun.,* Vol. COM-49, No. 1, 2001, pp.5–8.

[10] J.D. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, New York, 1995.

[11] J.H. Barnard and C.K. Pauw, "Probability of error for selection diversity as a function of dwell time", *IEEE Trans. Commun.*, Vol. COM-37, No. 8, 1989, pp. 800–803.

[12] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products.* 5th ed. Academic Press, New York, 1994.