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Network-assisted diversity for random access wireless sensor networks under the use of the generalized approach to signal processing

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ABSTRACT

A novel viewpoint based on the generalized approach to signal processing in the presence of noise and devoted to the collision resolution problem is introduced in this paper for wireless slotted random access sensor networks. Signal separation principles borrowed from signal processing problems are used. The received collided packets are not discarded in this approach but are exploited to extract each individual sensor node packet information. In particular, if k sensor nodes collide in a given time slot, they repeat their transmission for a total of k times so that k copies of the collided packets are received. Then the receiver has to resolve a $k \times k$ source-mixing problem and separate each individual sensor node. The generalized receiver does not introduce throughput penalties since it requires only k slots to transmit k colliding packets. In the course of analysis, we consider four channels models: ideal additive white Gaussian noise channel, in which the *i*-th sensor node's gain is a deterministic but unknown constant; non-fading channel with power control but arbitrary phase, in which the amplitude of the *i*-th sensor node's gain is constant (may be unknown), whereas the phase is random and uniformly distributed within the limits of the interval $[0,2\pi]$; Rayleigh fading channel, in which the phase is uniformly dist-

ributed within the limits of the interval $[0,2\pi]$, whereas the amplitude is distributed with the parameter σ_A , and the amplitude and phase are independent; Rician fading channel, in which the phase is uniformly distributed within the limits of the interval $[0,2\pi]$, whereas the amplitude is Rician distributed with the parameter A and σ_A , and the amplitude and phase are independent. Performance issues that are related to the implementation of the collision detection algorithm based on the generalized approach to signal processing in the presence of noise demonstrate a great superiority in comparison with well-known methods. The protocol's parameters are optimized to maximize the system throughput. Under the use of the generalized approach to signal processing in the presence of noise, the system throughput is higher in comparison with modern methods and algorithms.

Keywords: collision resolution, generalized receiver, diversity, multiple access, random access.

1. INTRODUCTION

The majority of wireless sensor network services provide a circuit switched and constant bit rate service to each sensor node. In these circumstances, multiplexing various sensor nodes may be accomplished with relatively simple TDMA, FDMA, or CDMA techniques.¹ There is, however, an increasing interest in wireless sensor data services and/or multimedia services, where variable bit rate sources have to be multiplexed. In this case, simple TDMA solutions are extremely inefficient, and some random access techniques are typically preferred.² Simple random access protocols of the ALOHA type offer a relatively straightforward implementation and can accommodate bursty sensor nodes. They suffer, however, from a severe throughput penalty and underutilization of the channel resources.³ Carrier sensing (CSMA) and /or collision detection mechanisms are typically employed in an effort to improve the throughput performance of random access schemes.³ In a wireless sensor network environment, however, collision detection features are not feasible due to the magnitude of the signal attenuation. Even carrier sensing may not be reliably performed due to unpredictability of the wire-

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less sensor network channel.¹ For this reason, in wireless sensor networks, data sensing is usually employed (DSMA).⁴ In this scheme, the sink detects collisions and continuously broadcasts a busy/idle signal in a control channel to all sensor nodes, providing the necessary feedback. DSMA, like other random access alternatives, suffers from relatively low throughput compared with the constant bit rate TDMA case. The reason is that channel resources are wasted when a collision occurs since the channel does not provide useful service during a collision. The emphasis in the random access literature

has been mostly on retransmission schemes that minimize future collisions.² However, when a collision does occur, the collided packets are typically discarded, and no information is exploited from them. It is clear that the throughput penalty incurred by collisions cannot be eliminated unless some way is devised to extract useful information from the collided packets. Little attention has been paid in the past to the approach of separating the collided transmissions instead of just discarding all collided packets. Fortunately, however, in the communications and signal processing literature, there has

recently been intense research activity in the area of user separation⁵ and interference rejection⁶, mostly in the context of space-division-multiple-access.⁷ This viewpoint, however, is virtually absent in medium access problems, due perhaps to the limited interaction between the networking and signal processing research communities. The goal of this paper is to

provide the use of the generalized approach to signal processing in the presence of noise⁸⁻¹² to the random medium access problem. In this approach, received packets that have collided are stored in memory rather than being discarded. They are later combined with future retransmissions in order to extract all the collided information packets. The technique exploits *diversity-combining* ideas in order to separate the collided packets. It differs however, from classical diversity methods since the required diversity is not created through multiple receiving antennas. Instead, wireless sensor network resources are used to provide diversity through selective transmissions. For this reason, this method is named the

network-assisted diversity multiple access (NDMA).¹³ Similar ideas of a network-wide collaboration have been investi-

gated in ¹⁴ in a different context. In that problem, the antenna patterns are jointly optimized. The main advantage of the proposed NDMA technique based on the generalized approach to signal processing in the presence of noise is that, firstly, no channel slot is lost when a collision takes a place and, secondly, the system throughput is higher in comparison with modern methods and algorithms in signal processing. If, for example, three sensor nodes collide, only three time slots are required to resolve the collision and successfully forward the three information packets. Since this is the same number of slots required if there were no collisions, the NDMA technique based on the generalized approach to signal processing in the presence of noise introduces no throughput penalties. Moreover, if a subset of *k* users experience heavy load and have unstable buffers, the system's performance gracefully degrades into the equivalent of a *k*-TDMA system. Hence, the proposed NDMA technique based on the generalized approach to signal processing in the presence of noise is suitable for multiplexing variable-bit-rate sources without affecting the physical layer bit rate parameter of each sensor node. ¹⁵

2. PROBLEM STATEMENT

Consider a wireless random access sensor network system with K sensor nodes. The system is slotted, and at each time slot n, each sensor node i may transmit a packet consisting of N symbols

$$\mathbf{a}_{N,i}(n) = [a_{n,i}(1), \dots, a_{n,i}(N)]^T,$$
(1)

provided that its packet buffer is nonempty (see Fig.1). The symbols $a_{n,i}(l), l = 1, 2, ..., N$ are assumed drawn from a finite constellation. The received baseband discrete-time signal at the input of the generalized receiver, i.e., at the output of linear tract of the generalized receiver (the output of the preliminary filter), after sampling at the symbol rate is

$$y_n(l) = \sum_{i \in I(n)} \alpha_i(n) a_{n,i}(l) + v_n(l), \quad l = 1, 2, ..., N$$
(2)

where I(n) is the index set of sensor nodes that are active at time slot n; $v_n(l)$ is the additive noise at the output of the preliminary filter of the generalized detector; $\alpha_i(n)$ is the *i*-th sensor node's gain. A non-frequency-selective channel is considered in this paper, and therefore, the gains are

$$\alpha_i(n) = A_i(n) \cdot e^{j\varphi_i(n)} , \qquad (3)$$

where $A_i(n)$ and $\varphi_i(n)$ denote the amplitude and phase, respectively. The following assumptions will be made regarding

the model of (1): $v_n(l)$ is zero mean, complex circular additive Gaussian noise with the variance σ_v^2 in *n* and *l*, and $\varphi_i(n)$ is a uniform $[0,2\pi]$ phase, independent and identically distributed in *i* and *n*. Notice that *n* is assumed independent and identically distributed from packet to packet and not from bit to bit. The amplitude $A_i(n)$ may be assumed to be either constant or randomly distributed, depending on whether the channel is fading and whether power control is implemented or not. Furthermore, each sensor node *i* is assumed to have an infinite length buffer that holds fixed length data packets. If we collect all measurements within a time slot (l = 1, ..., N) in a vector

$$\mathbf{y}_{N}(n) = [y_{n}(1), \dots, y_{n}(N)]^{T}$$
(4)

and similarly for $\mathbf{v}_{N}(n)$, then (1) may be written as





$$\mathbf{y}_{N}(n) = \sum_{i \in I(n)} \alpha_{i}(n) \cdot \mathbf{a}_{N,i}(l) + \mathbf{v}_{N}(l) .$$
(5)

It is customary to discard a packet $\mathbf{y}_N(n)$ when a collision is detected and initiate a retransmission schedule. As can be seen from (5), however, $\mathbf{y}_N(n)$ contains information about the transmitted packets and should be exploited. Before we explore collision resolution ideas based on (5), we conclude this section with a short discussion of the shortcomings of (5). The most restrictive assumption is the perfect synchronization of all sensor nodes implicit in (5). That restricts its applicability to wireless sensor network systems with synchronization control (sensor nodes may be synchronized only for one particular receiver location), and therefore, the notion of a sink becomes necessary. Furthermore, no multipath effects are taken into account in (5). Asynchronous sensor nodes and dispersive channels are outside the scope of the present paper.

3. COLLISION RESOLUTION THROUGH SIGNAL SEPARATION

Review shortly the main ideas discussed in.¹³ Let us consider the case where k sensor nodes collide in time slot n; therefore, $l(n) = \{i_1, ..., i_k\}$. Then, $\mathbf{y}_N(n)$ consists of a mixture of k sources that need to be separated. From a signal processing viewpoint, this problem may be solved if we are able to create a K-branch diversity, i.e., using K antennas, with $K \ge k$ and collect K independent mixtures of the signals $\mathbf{a}_{N,i}$ according to space division multiple access techniques.⁶ In the current random access framework, however, the interesting question is whether we can utilize wireless sensor network resources at the protocol level to create the necessary diversity without multiple antennas. The answer to this question is in the affirmative, as explained next. Assume for the moment that all sensor nodes are aware of, firstly, whether there has been a collision at time slot n and, secondly, its multiplicity k. Assume furthermore that according to the protocol, each of the k colliding sensor nodes will retransmit its information packet k - 1 more times in th next k - 1 slots, i.e., in slots n + 1..., n + k - 1. Finally, no other sensor node will initiate a new transmission in the next k - 1 slots. An example of this procedure for a collision of two sensor nodes is shown in Fig. 2a. With these conventions, the sink will receive a total of k copies of the collided packets

$$[\mathbf{y}_{N}(n),...,\mathbf{y}_{N}(n+k-1)] = [\mathbf{a}_{N,i_{1}}(n),...,\mathbf{a}_{N,i_{k}}(n)] \cdot \begin{vmatrix} \alpha_{i_{1}}(n)\Lambda & \alpha_{i_{1}}(n+k-1) \\ \mathbf{M} & \mathbf{O} & \mathbf{M} \\ \alpha_{i_{k}}(n)\Lambda & \alpha_{i_{k}}(n+k-1) \end{vmatrix} + [\mathbf{v}(n),...,\mathbf{v}(n+k-1)] , \qquad (6)$$

or equivalently

$$\mathbf{Y}_{N,k}(n) = \mathbf{A}_{N,k}(n) \cdot \mathbf{A}(n) + \mathbf{V}_{N,k}(n)$$



Fig. 2. Packet collision and retransmission.

with obvious definitions for the matrices in (7). Equation (7) represents a classical source separation problem. If the mixing matrix A(n) is known or can be estimated, then the maximum likelihood estimate of the transmitted packets can be shown to be

$$\hat{\mathbf{A}}_{N,k}(n) = \arg\min_{\mathbf{A}} \left\| \mathbf{Y}_{N,k}(n) - \mathbf{A} \cdot \boldsymbol{A}(n) \right\|_{F}^{2},$$
(8)

where $\| ... \|_F$ represents the Frobenius-2 norm, and A takes all possible finite values, depending on the signal constellation. The solution (8) has exponentially increasing complexity in the number of colliding sensor nodes and may be impractical. Suboptimal linear solutions can readily be employed as

$$\hat{\mathbf{A}}_{N,k}(n) = \mathbf{Y}_{N,k}(n) \cdot \boldsymbol{A}^{-1}(n)$$
(9)

provided that A(n) has full rank. The latter property of A(n) is guaranteed under assumptions made regarding the model of (1). Notice that only k slots are required to make A(n) square and, therefore, resolve k colliding sensor nodes. Hence, no slots are wasted, and no throughput penalties are incurred by this collision resolution method. Now, we return to the assumption requiring all sensor nodes to be aware of the collision. In a wireless sensor network environment, it is natural that the sink identifies that information and broadcasts it back in a separate control channel, for example, the DSMA pro-

tocol.¹ In the next section, we will develop a collision detection procedure for the current setup, which can be implemented by the sink. This procedure also produces the estimate of the mixing matrix A(n) used for recovering the data packets. As far as the feedback procedure is concerned, a simple implementation is using a broadcast control channel with rate one bit/time slot. At the end of slot n, the sink may indicate in the control channel whether the next slot is free or busy, i.e., reserved for retransmission. If slot n+1 is indicated to be busy, all sensor nodes that transmitted in slot n will retransmit. If the sink repeats the busy indication for k-1, then slots that are all colliding sensor nodes will be forced to retransmit k - 1 times. Of course, new sensor nodes will not be allowed to transmit while the busy signal is on. The retransmission can also be arranged in an orderly fashion if a more complex control channel is available. Since the sink is able to detect a collision and determine the identities of the colliding sensor nodes at slot n, it may reserve the slot n + 1 for a specific sensor node to retransmit by broadcasting some instruction information on the control channel to inform that sensor node to retransmit and prohibit other sensor nodes to either retransmit or start a new transmission. With a k-multiplicity collision, i.e., $I(n) = \{i_1, ..., i_k\}$, the sink may instruct sensor node $i_2, ..., i_k$ to retransmit one by one in the following k-1slots, and at the end of slot n + k-1, with the data from other k - 1 sensor nodes at hand, the sink can subtract the interference from those k-1 sensor nodes from the first colliding transmission to recover the packet of the sensor node i_1 . Figure 2b shows such a retransmission procedure for a collision of two sensor nodes. In fact, this retransmission scheme is different from the one discussed in the previous paragraph only in that it produces a different mixing matrix A(n) from that in (7)

(7)

$$\boldsymbol{A}(n) = \begin{vmatrix} \alpha_{i_1}(n) & \alpha_{i_2}(n+1) & \Lambda & \alpha_{i_1}(n+k-1) \\ 0 & \alpha_{i_2}(n+1) & \Lambda & 0 \\ M & M & O & M \\ 0 & 0 & \Lambda & \alpha_{i_k}(n+k-1) \end{vmatrix}.$$

A mixing matrix of such a structure makes the collision resolution procedure somewhat simpler at the expense of a more complex control channel. Following this train of thought, we can also derive a number of other retransmission protocols that result in different mixing matrices; however, the same resolution techniques described in (8) and (9) also apply to them. The last comment on the collision resolution procedure is that there exists the possibility that the square-mixing matrix A(n) obtained through retransmissions may accidentally lose rank, and thus, the collision resolution method fails.

In this case, the sink may continue requesting retransmissions more than the collision multiplicity k, until A(n) is full rank. The collision resolution techniques in (8) and (9) are still valid, except that the inverse operation in (9) should be changed to LS inverse. Further simplification is possible at the expense of some throughput penalties. For example, the sink may instruct the colliding sensor nodes to retransmit one at a time over the next k slots in an orderly fashion; hence, the sensor node separation step can be omitted. This approach is simpler at the expense of one wasted slot out of k + 1slots. The first collided transmission is discarded. Now we pay our attention to the problem of how the sink may identify the colliding sensor nodes.

4. DETECTION TECHNIQUE OF COLLISION

For every time slot n, the sink constructed on the basis of the generalized approach to signal processing in the presence of the noise has to identify the set of active sensor nodes l(n). In other words, it has to make a decision for each sensor no-

de *i* as to whether it is active or not. Therefore, there are a total of 2^{K} different possibilities in a *K*-sensor node system. In order for the sink to discriminate the sensor nodes, an address field is required in the packet that contains a unique ID sequence for each sensor node. Let us assume without loss of generality that the first *M* symbols of each packet of sensor node *i* form an identifying vector **a**.

$$\mathbf{a}_i = [\mathbf{a}_{N,i}(n)]_{1:M} \quad . \tag{11}$$

(10)

In addition, let

$$\mathbf{y}(n) = \left[\mathbf{y}_{N}(n)\right]_{1:M} , \qquad (12)$$

and

$$\mathbf{v}(n) = \left[\mathbf{v}_{N}(n)\right]_{1:M} \quad \text{and} \quad \mathbf{v}^{*}(n) = \left[\mathbf{v}^{*}_{N}(n)\right]_{1:M} , \qquad (13)$$

where $\mathbf{v}(n)$ is the noise at the output of the preliminary filter of the generalized detector and $\mathbf{v}^*(n)$ is the additional (reference) noise at the output of the additional filter of the generalized detector, which is uncorrelated with the noise $\mathbf{v}(n)$ and has the same statistical parameters as the noise $\mathbf{v}(n)$, since the noise $\mathbf{v}(n)$ and $\mathbf{v}^*(n)$ are obtained at the input of the generalized detector from the common noise; in a general case, the statistical parameters of the noise $\mathbf{v}(n)$ and $\mathbf{v}^*(n)$ are differed. Based on (5)

$$\mathbf{y}(n) = \sum_{i \in I(n)} \alpha_i(n) \mathbf{a}_i + \mathbf{v}(n) \Longrightarrow H_{i,1} \quad \text{and} \quad \mathbf{y}^*(n) = \mathbf{v}^*(n) \Longrightarrow H_{i,0} \quad .$$
(14)

Equation (14) reveals the fact that the estimation $\alpha_i(n)$ from the data $\mathbf{y}(n)$ is a linear regression problem, whereas the identification of I(n) is a model (or regressor) selection problem. A simple comparison of the modeling error for each hypothesis is not sufficient since it is well known that the error always decreases as more regressors are added. ¹⁶ However, most classical model selection procedures are applicable to this problem. ^{17–19} Unfortunately, due to the large number of hypothesis (2^{κ}), all those procedures will be computationally demanding and their complexity will increase exponential-

ly with the number of the sensor nodes. In order to simplify the collision detection procedure, in the sequel, we make the following assumption: the ID sequences are orthogonal to each other

$$\mathbf{a}_{i}^{H}\mathbf{a}_{k} = \begin{cases} 0 & i \neq k, \\ 1 & i = k. \end{cases}$$
(15)

The unit norm assumption implied here can be made without loss of generality. Under this assumption, it can be shown that the joint detection problem can be decoupled into K independent single sensor node-detection problems. Indeed, by left multiplying with \mathbf{a}_i^H both sides of (14), we may get

$$z_{i}(n) = \begin{cases} \mathbf{a}_{i}^{H} \mathbf{y}(n) = \alpha_{i}(n) + \mathbf{a}_{i}^{H} \mathbf{v}(n) \Rightarrow H_{i,1}, \\ \mathbf{a}_{i}^{H} \mathbf{y}^{*}(n) = \mathbf{a}_{i}^{H} \mathbf{v}^{*}(n) \Rightarrow H_{i,0}. \end{cases}$$
(16)

In addition, $z_i(n)$ is the statistic at the input of the generalized detector (at the output of linear tract of the generalized detector) for detecting sensor node *i*. Then, the conditional probability distribution densities for sensor node *i*, $H_{i,0}$ corresponding to *i* not in *l* (*n*) and $H_{i,1}$ corresponding to *i* in *l* (*n*) according to the generalized approach to signal processing in the presence of noise^{8–12} are

$$f[\mathbf{y}^{*}(n) | H_{i,0}] = \frac{1}{\sqrt{2\pi\sigma_{\nu}^{2}}} \exp\left(-\frac{|z_{i}(n)|^{2}}{2\sigma_{\nu}^{2}}\right), \qquad (17)$$

$$f[\mathbf{y}(n) | \alpha_i(n), H_{i,1}] = \frac{1}{\sqrt{2\pi\sigma_v^2}} \exp\left(-\frac{|z_i(n) - \alpha_i(n)|^2}{2\sigma_v^2}\right) , \qquad (18)$$

The likelihood ratio is

$$\Lambda[\mathbf{y}(n) | \alpha_{i}(n)] = \frac{f[\mathbf{y}(n) | \alpha_{i}(n), H_{i,1}]}{f[\mathbf{y}^{*}(n) | H_{i,0}]} = \exp\left[\frac{2z_{i}(n)\alpha_{i}(n) - z_{i}^{2}(n) + \mathbf{v}^{*^{2}}(n)}{2\sigma_{v}^{2}}\right].$$
(19)

The generalized detector for sensor node *i* depends on the statistical properties of $\alpha_i(n)$. According to different channels models, $\alpha_i(n)$ falls into four cases.

- Ideal additive white Gaussian noise channel: $\alpha_i(n)$ is a deterministic but unknown constant.
- Non-fading channel with power control but arbitrary phase: The amplitude of $\alpha_i(n)$ is constant, may be even unknown, $A_i(n) = A$, whereas the phase $\varphi_i(n)$ is random and uniformly distributed within the limits of the interval $[0,2\pi]$.
- *Rayleigh fading channel:* The phase $\varphi_i(n)$ is uniformly distributed within the limits of the interval $[0,2\pi]$, whereas the amplitude $A_i(n)$ is Rayleigh distributed with the parameter σ_A^2 , and $A_i(n)$ and $\varphi_i(n)$ are independent.
- *Rician fading channel:* The phase $\varphi_i(n)$ is uniformly distributed within the limits of the interval $[0,2\pi]$, whereas the amplitude $A_i(n)$ is Rician distributed with the parameter σ_A^2 , and $A_i(n)$ and $\varphi_i(n)$ are independent.

Of the following four cases, the first one violates the first assumption made under definition of the model (1). It is included, however, for completeness. For these cases, the generalized detector defines the maximum likelihood test, which is estimating $\alpha_i(n)$ by maximizing the likelihood function, the plugging the estimate into (19) and comparing the result with a threshold K_g . Despite the different distributions of $\alpha_i(n)$, it turns out that the generalized detector under all the four channel environments is the same ⁸⁻¹²

$$2z_i(n)\alpha_i(n) - z_i^2(n) + \mathbf{v}^{*2}(n) > K_g \Longrightarrow H_{i,1} \quad \text{or} \quad 2z_i(n)\alpha_i(n) - z_i^2(n) + \mathbf{v}^{*2}(n) < K_g \Longrightarrow H_{i,0} \quad , \tag{20}$$

i.e., we need to compare an amplitude of the output of the generalized detector matched to the ID sequence a, with a pre-

determined threshold K_g . In addition, the probability of false alarm P_F under the four channel environments is also the same

$$P_{F} = \int_{K_{g}}^{\infty} \frac{1}{2\pi\sigma_{v}^{2}} K_{0}\left(\frac{x}{2\sigma_{v}^{2}}\right) dx , \qquad (21)$$

where $K_0(x)$ is the modified second kind Bessel function of an imaginary argument or, as it is called, McDonald's function on and $\mathbf{x} = \mathbf{v}^{*^2}(n) - \mathbf{v}^2(n)$. The probability of detection P_D under each channel environment is listed below.

Ideal additive white Gaussian noise channel

$$P_{D} = \int_{K_{g}}^{\infty} \frac{1}{2\pi\sigma_{v}^{2}} K_{0} \Big(-\frac{v^{*^{2}}(n) - v^{2}(n) + A_{i}^{2}(n)}{2\sigma_{v}^{2}} \Big) d[v^{*^{2}}(n) - v^{2}(n)] .$$
(22)

Non-fading channel with power control but arbitrary phase:

$$P_{D} = \int_{K_{g}}^{\infty} \frac{1}{2\pi\sigma_{v}^{2}} K_{0} \left(-\frac{v^{*}(n) - v^{2}(n) + A^{2}}{2\sigma_{v}^{2}} \right) d[v^{*}(n) - v^{2}(n)] .$$
⁽²³⁾

Rayleigh fading channel:

$$P_{D} = \int_{K_{g}}^{\infty} \frac{1}{2\pi(\sigma_{v}^{2} + \sigma_{A}^{2})} K_{0} \Big(-\frac{v^{*2}(n) - v^{2}(n) + A_{i}^{2}(n)}{2(\sigma_{v}^{2} + \sigma_{A}^{2})} \Big) d[v^{*2}(n) - v^{2}(n)] .$$
(24)

• Rician fading channel:

$$P_{D} = \int_{K_{g}}^{\infty} \frac{1}{2\pi(\sigma_{v}^{2} + \sigma_{A}^{2})} K_{0} \left(-\frac{v^{*^{2}}(n) - v^{2}(n) + A^{2}}{2(\sigma_{v}^{2} + \sigma_{A}^{2})} \right) d[v^{*^{2}}(n) - v^{2}(n)] .$$
(25)

Another point that deserves notice is that the statistic $z_i(n)$ also gives us the maximum likelihood estimate of $\alpha_i(n)$. This can be easily proved by maximizing (18) with respect to $\alpha_i(n)$. Hence, given estimates of all $\alpha_i(n)$, i.e., I(n), we may compose the mixing matrix A(n) and recover the sensor node data packets according to (9). Show the role of the probability of false alarm P_F and the probability of detection P_D in the expression of the wireless network system throughput and optimize them to maximize the wireless network system throughput.

5. PERFORMANCE ANALYSIS

As was discussed in Section 3, extra slots are not required to resolve a packet collision and, therefore, there are no throughput penalties for the considered wireless sensor network system. This conclusion, however, is correct only under assumption that the sink makes no errors in identifying the number of active sensor nodes in every slot. For example, if two sensor nodes collide but the sink incorrectly concludes that only one sensor node is present, no retransmission will be requested, and the collision will not be resolved. It is, therefore, evident that in order to more accurately assess the performance of the considered wireless sensor network systems, we need to study the probability of incorrect sink decisions and their effects on the wireless sensor network system's throughput. It is possible to devise a number of different variations of the considered approach, depending on whether the sink is allowed to correct its initial estimate of the number of active sensor nodes based on subsequent transmissions. For instance, two sensor nodes may be detected in the first transmission and three sensor nodes may be detected in the second transmission, in which case, the sink may either retain or modify its original estimate. Other variations may arise if we consider different retransmission schedules for packets that were incorrectly resolved. Finally, the analysis is further complicated by the fact that an incorrect sink decision may not necessary imply that the packets are lost. For example, if two packets collide but the sink decides that three collided, and then the two original packets will eventually be resolved. In order to provide a unified analysis framework for all those cases, we will take a pessimistic approach and study the worst-case scenario. We will assume that, firstly, the sink is not

allowed to correct its original decision on the number of active sensor nodes, and, secondly, every incorrect decision by the sink results in the loss of all packets involved in that transmission epoch. Finally, we will not be concerned here with packets that are resolved but lost due to excessive bursts of errors in the payload portion of the packets. These losses are not due to the random access protocol but due to inadequate error correcting coding. It will be instructive for our analysis purposes to view the traffic in the channel as a flow of collision resolution periods or epochs. An epoch includes one or several consecutive channel slots that are dedicated for the transmission (including the initial transmission and the retransmissions) of the data packets from the sensor nodes that are active at the beginning of the epoch. The idle slots, during which no data are transmitted, also compose epochs called idle epochs, which only include one slot. Correspondingly, we call those epochs, during which some packets are under transmission, busy epochs. The length of a busy epoch is the number of time slots the channel takes to serve the currently active sensor nodes. From a throughput viewpoint, it is important to further distinguish busy epochs into useful epochs, meaning absence of detection errors, and corrupted epochs, meaning presence of detection errors. Only useful epochs contribute to the channel throughput. The epoch flow in the channel is shown clearly in Fig. 3. The epoch length is a random variable depending on the number of the active sensor nodes at the beginning of the epoch. Denote by P_e the probability of a sensor node's buffer being empty at the beginning of an epoch. Then, we can obtain binomial expressions for the probability of the epoch length:

$$P_{busy}(k) = \binom{K}{k} \cdot (1 - P_e)^k P_e^{K-k}, \quad k = 1, 2, ..., K$$
(26)



Fig. 3. Epoch flow and embedded Markov points.

$$P_{idle}(k) = \begin{cases} P_e^K, & k = 1\\ 0, & \text{otherwise.} \end{cases}$$
(27)

The expressions are only an approximation of the distribution of epoch length in the generalized approach to signal processing environment when the detection errors have minor effects on the traffic in the channel. Finally, we can write

 $P_{useful}(k) = P_{busy}(k) \cdot P(\text{correct detection} \mid k \text{ active sensor nodes})$

$$= \binom{K}{k} \cdot (1 - P_e)^k P_e^{K-k} P_D^k (1 - P_F)^{K-k}.$$
(28)

With these preliminary developments, we can define the throughput expression. Let us define the throughput as

Under this definition, using (26) and (28), we can write

C

$$C = \frac{\sum_{k=1}^{K} k \binom{K}{k} \cdot (1 - P_e)^k P_e^{K-k} P_D^k (1 - P_F)^{K-k}}{\sum_{k=1}^{K} k \binom{K}{k} \cdot (1 - P_e)^k P_e^{K-k} + 1 \cdot P_e^K}$$
(30)

The denominator in (30) is immediately $K(1-P_e) + P_e^K$. The numerator is recognized as the binomial expansion of

(29)

$$x\left(\frac{\partial}{\partial x}\right) \cdot \left(x+y\right)^{K} = Kx(x+y)^{K-1}$$
(31)

with

$$x = (1 - P_e)P_D$$
 and $y = P_e(1 - P_D)$. (32)

Finally, we can write

$$C = \frac{K(1 - P_e)}{K(1 - P_e) + P_e^K} \cdot P_D \left[(1 - P_e) P_D + P_e (1 - P_F) \right]^{K-1} .$$
(33)

As signal-to-noise ratio tends to approach infinity, we expect $P_D \rightarrow 1$ and $P_F \rightarrow 0$. Consequently, the throughput can be determined in the following form:

$$C \to \frac{K(1-P_e)}{K(1-P_e) + P_e^K} . \tag{34}$$

As we will see shortly, if every sensor node's buffer is fed with a Poisson source with density λ , the right-hand side of (34) also equals to λK , which is total offered traffic rate. According to (33), the throughput depends on the probability of detection P_D and probability of false alarm P_F , which in turn depend on the threshold K_g of the generalized detector. It is therefore worthwhile to appropriately select the threshold K_g so that the throughput is maximized. Since the threshold K_g is one to one with the probability of false alarm P_F according to (21), we may equivalently obtain the optimal value of the probability of false alarm P_F . The solution is obtained by setting the derivative

$$\frac{dC}{dP_F} = 0 \quad . \tag{35}$$

Naturally, the probability of detection P_D is a function of the probability of false alarm P_F , as given by receiver operation curve of the generalized detector. We therefore obtain a solution in terms of $\frac{dP_D}{dP_F}$. After some tedious but straightforward differentiation, we obtain the relation

$$\frac{dP_D}{dP_F} = \frac{K - 1}{K \frac{1 - P_e}{P_e} + \frac{1 - P_F}{P_0}} \,. \tag{36}$$

Depending on the particular fading conditions assumed and, therefore, the particular receiver operation curve expressions of the generalized detector given in Section 4, (36) can be solved for the optimal value of the probability of false alarm P_F and further for the optimal threshold K_g of the generalized detector. At high values of the signal-to-noise ratio, we can simplify (36) by the following approximation

$$\frac{1-P_F}{P_D} \approx 1 . \tag{37}$$

Then, (36) becomes

$$\frac{dP_D}{dP_F} = \eta \quad , \tag{38}$$

$$\eta = \frac{K - 1}{1 + K \frac{1 - P_e}{P_e}} \,. \tag{39}$$

Notice that the traffic load parameter η depends only on the traffic characteristics and not on the signal-to-noise ratio or parameters of the generalized detector. Solve (38) and (39) for the case of the Rayleigh fading channel. In this case, the probability of false alarm P_F and the probability of detection P_D are given by (21) and (24), respectively. Defining a function between the probability of detection P_D and the probability of false alarm P_F , determining $\frac{dP_D}{dP_F}$ and the optimal probability of false alarm P_F , we can obtain the optimal threshold of the generalized detector

$$K_g^{opt} = 2\sigma_v^2 \ln \eta \tag{40}$$

Notice that the optimal threshold of the generalized detector monotonically increases with the traffic load parameter η and is independent of the signal-to-noise ratio. This is a great peculiarity of the generalized detector. It is evident from (40) that the optimal threshold depends on the traffic characteristics. In particular, it depends on the probability of a sensor node's buffer being empty, P_e (through η). It is therefore important to evaluate P_e given a traffic load λ . Focus our attention on the beginning instant of each epoch (see Fig. 3) and denote q_m as the number of data packets in the buffer of a sensor node at the beginning time instant of the *m*-th epoch. The time index *m* counts in epochs and not slots. The sequence q_m constitutes an embedded Markov chain. Define the expression for $\Pr\{q_m = k\}$. As a special case,

$$P_e = \lim_{m \to \infty} \Pr\{q_m = k\} \quad . \tag{41}$$

From the viewpoint of a particular sensor node, two types of epochs can be distinguished: relevant epochs, in which a data packet belonging to this sensor node is being transmitted, and irrelevant epochs, in which no packet belonging to this user is being transmitted (see Fig. 3). In other words, at the beginning of an irrelevant epoch, the buffer of this specific sensor node is empty. The length of the two types of epochs, which are denoted h_1 and h_2 , respectively, obey different distributions

$$P_{h_1}(k) = \binom{K-1}{k-1} \cdot (1-P_e)^{k-1} P_e^{K-1}, \quad 1 \le k \le K \quad ;$$
(42)

$$P_{h_2}(k) = \begin{cases} P_e^{K-1} + (K-1) \cdot (1-P_e) P_e^{K-2}, & k = 1; \\ \binom{K-1}{k} \cdot (1-P_e)^k P_e^{K-k-1}, & 1 < k \le K-1. \end{cases}$$
(43)

Equations (42) and (43) are accurate only if the sink makes no detection errors. Use them in our analysis, however, as approximate distributions of the relevant and irrelevant epoch length under high signal-to-noise ratio. Let $v(q_m)$ be the number of data packets arriving during the *m*-th epoch will be an irrelevant epoch for this sensor node, whereas if $q_m > 0$, the *m*-th epoch will be a relevant epoch, and thus, $v(q_m)$ will obey different distributions. According to the state transition of the studied Markov chain, we obtain

$$q_{m+1} = \begin{cases} q_m - 1 + v(q_m), & q_m > 0; \\ v(q_m), & q_m = 0. \end{cases}$$
(44)

Denote by $Q_m(z)$ the probability generating function of q_m

$$Q_m(z) = \sum_{k=0}^{\infty} \Pr\{q_m = k\} z^k = E[z^{q_m}]$$
(45)

and $Q(z) = \lim_{m \to \infty} Q_m(z)$ the steady state of $Q_m(z)$. Similarly,

$$F(z) = \lim_{m \to \infty} E[z^{\nu(q_m)} | q_m = 0] \quad \text{and} \quad G(z) = \lim_{m \to \infty} E[z^{\nu(q_m)} | q_m > 0] .$$
(46)

As was shown in ¹³, using those definitions, we may present the following result. If the sensor node's buffer is fed by a Poisson source with rate λ , the steady-state probability generating function Q(z) is given by

$$Q(z) = P_e \frac{zF(z) - G(z)}{z - G(z)} ,$$
(47)

where $G(z) = \sum_{k=1}^{K} e^{k(z\lambda-\lambda)} P_{h_1}(k)$ and $F(z) = \sum_{k=1}^{K-1} e^{k(z\lambda-\lambda)} P_{h_2}(k)$. Evaluating (47) at z = 1, we can obtain a relationship

between P_e and λ . As was discussed in ¹³, the probability P_e is the unique solution in [0,1] of the equation

$$\lambda P_e^K + (1 - \lambda K)P_e - (1 - \lambda K) = 0 \quad . \tag{48}$$

Let us remark at this point that (48) has indeed a unique solution in [0,1] since the polynomial $D(P_a) = \lambda P_a^K + (1 - \lambda K)$

 $\times P_e - (1 - \lambda K)$ has the properties $D(0) = -(1 - \lambda K) < 0$, $D(1) = \lambda > 0$, and $D'(P_e) = \lambda K P_e^{K-1} + (1 - \lambda K) > 0$. From (48), it is easy to derive the equation

$$\frac{K(1-P_e)}{P_e^K + K(1-P_e)} = \lambda K \quad . \tag{49}$$

Substituting (49) into (33), we get an intuitive result that the throughput equals the total offered traffic times the probability of correct detection.

6. COMPUTER SIMULATION RESULTS

Advantages of NDMA technique in wireless networks over TDMA approach were discussed in ¹³. Show some illustrative plots of our analysis results. The plots highlight the advantages of the use of the generalized detector in NDMA for wireless sensor networks in comparison with the use of the optimal receivers of modern signal processing in terms of throughput. In plotting the analytical expressions (33), we set the wireless sensor network systems parameter as follows. The sensor nodes population K = 16, each sensor node has an infinite buffer, and the signal-to-noise ratio is 5 and 10 dB (see Fig. 4a and b, respectively). Sensor nodes' ID sequences were selected form K-th order Hadamard matrix. The sensor node packets were fixed length of N = 424 bits. CRC codes were used for error detection. All sensor nodes maintained packet buffers that were fed by Poisson sources with density λ , generating data packets to contribute to the wireless sensor network traffic. The channel between each sensor node and the sink was Rayleigh fading. We plot the throughput C as function of the total traffic load λK . Figure 4 illustrates C versus λK , respectively. In Fig. 4, a diagonal dotted line is plotted that represents the throughput of the TDMA approach. As we know, under the infinite buffer assumption, there is no packet loss for the TDMA wireless sensor network system; hence, its throughput is perfectly equal to the offered load. We may see that the use of the generalized detector in NDMA wireless sensor network system offers the same throughput performance as the TDMA, even in very heavy traffic load. The advantage of the use of the generalized detector in NDMA for wireless sensor network system in comparison with the optimal detectors of modern signal processing is manifested by comparison in Fig.4.

7. CONCLUSIONS

Under the use of the generalized detector in NDMA for wireless sensor network system, the collision detection is only performed once for each transmission epoch at the first slot. We do not allow the sink to modify its decision in the following retransmission slots. Therefore, the statistic for detection is conditioned on the information received during the first slot. In fact, the generalized detector should exploit all the information received from the beginning up to the present. It is not hard to modify the generalized detector to include all information received from the first transmission up to the k-th retransmission, denoting the first collided transmission as the zero-th retransmission. Since the received noise contribution is independent for each retransmission, the sufficient statistic for the sensor node i on the k-th retransmission received

is just the summation of the sufficient statistic of each of the k + 1 slots (see (16)) $z_i(n,k) = \sum_{l=0}^k \alpha_i(n+l) + \mathbf{a}_i^H \sum_{l=0}^k \mathbf{v}(n+l)$

(+ l). The generalized detector is still the comparison of the amplitude of $z_i(n,k)$ with a threshold

$$|z_i(n,k)| > K_g(k) \Longrightarrow H_{i,1} \quad \text{and} \quad |z_i(n,k)| < K_g(k) \Longrightarrow H_{i,0} \quad .$$
(50)

The only change is that the threshold becomes a variable, depending on the retransmission count index k. The decision made by the generalized detector (50) is used to determine whether or not another (the (k + 1) st) retransmission is needed. If the number of sensor nodes detected as active according to (50) is larger than k + 1, the diversity is not enough, and more retransmissions will be required. If, conversely, the number of sensor nodes detected as active is less than or equal to k + 1, no more retransmissions will be required, and the sink begins to revoke the collision resolution procedure. In this paper, we studied the relatively simple scenario of flat fading environment. It is of more practical interest however to study the implementation of the generalized detector in NDMA for wireless sensor network system in the multipath fading channel environment. This is a colored signal separation problem, which is solvable under certain assumptions. This is one of interesting future research topic. The used collision detection approach under the use of the generalized detector



Fig. 4. Throughput versus traffic load.

in NDMA for wireless sensor network system requires orthogonality of the sensor node ID's. This, in turn, implies that the length of the ID sequences is required to increase linearly in the number of the sensor nodes and not logarithmically as most addressing schemes do. In the case of very large sensor node populations, this can create a severe drawback by effectively reducing the available bandwidth for carrying the packet payload. A solution to this problem may be blind collision detection and resolution. This is another interesting future research topic.

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