# Multiuser Generalized Detector for Uniformly Quantized Synchronous

## CDMA Signals in Wireless Sensor Networks with Additive White Gaussian Noise

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**Abstract**: We investigate the multiuser generalized detector, which is constructed based on the generalized approach to signal processing in noise [1]–[5], for uniformly quantized synchronous code division multiple access (CDMA) signals in additive white Gaussian noise channels of wireless sensor networks and compare with minimum mean-square error (MMSE) multiuser receiver discussed in [6]. The input/output relationship of the quantizer is represented by the gain plus the additive noise model. Based on this model, we derive the weight vector and the output signal-to-interference ratio (SIR) of the multiuser generalized detector. The effects of quantization and sampling of the multiuser generalized detector performance is characterized in a single parameter named "equivalent noise variance" which is a function of the sum of each active user's signal-to-noise ratio (SNR), processing gain, and the number of quantization and sampling levels. The optimal quantizer step size, which maximizes the multiuser generalized detector output SNR, is also determined. Simulation results validate the accuracy of our theoretical analysis and confirms a superiority of employment of the multiuser generalized detector over minimum mean-square error multiuser receiver, which is analyzed in [6].

**Keywords:** Multiuser generalized detector, additive white Gaussian noise channel, code division multiple access, quantization, wireless sensor network.

## **1. INTRODUCTION**

Multiuser detection has been demonstrated to be an effective way to mitigate the multiple access interference (MAI) and solve the near-far problem in CDMA systems [7]. Among the various multiuser detection schemes, the linear minimum mean-square error multiuser receiver is a popular candidate for its desirable compromise between performance and computational complexity [6]. We suggest to use the multiple generalized detector designed based on the generalized approach to signal processing in noise [1]–[5] and compare by performance the proposed multiuser generalized detector and the minimum mean-square error multiuser receiver [6].

In a digital CDMA multiuser generalized detector, as well as in a digital minimum mean-square error multiuser receiver, the incoming continuous signal is sampled and then quantized by the analog-to-digital converter (ADC). In general, the hardware complexity of the analog-to-digital converter is proportional to the number of bits used to represent the analog-to-digital converter output. Hence, it is desirable to minimize the bit requirement in the design of digital receivers. On the other hand, if the quantization is too coarse, the performance of both the multiuser generalized detector and minimum meansquare error multiuser receiver may degrade significantly due to quantization errors.

There have been a number of studies concerned with characterizing the effects of data quantization and sampling on the performance of CDMA signal detection [8]–[10]. However, none of the aforementioned papers considered how to analyze and design the minimum mean-square error multiuser receiver for the uniformly quantized CDMA signals based on the gain plus the additive noise model [11]. Only the paper [6] discusses this problem. Because of this, in the present paper we follow by

procedure described in [6] applying to design the multiuser generalized detector for uniformly quantized CDMA signals and compare performance of signal detection for the multiuser generalized detector designed based on the generalized approach to signal processing in noise [1]–[5] and for the minimum mean-square error multiuser receiver discussed in [6].

Since the operation of quantization and sampling is nonlinear, the weight vector of the multiuser generalized detector derived directly from the quantizer outputs is complicated. To simplify the analysis and design problems, we employ the gain plus additive noise model to represent the output of quantizer. With this simple but accurate model, we derive the weight vector of the multiuser generalized detector and optimize the signal-tointerference ratio of the multiuser generalized detector output for quantized CDMA signals. The effects of quantization and sampling on the multiuser generalized detector performance can be easily characterized in a single parameter named "equivalent noise variance".

In the present paper we use the following notations:  $(\cdot)^T$  and  $M(\cdot)$  stand for matrix transpose and expectation, respectively. The symbol I represents the identity matrix with proper dimension, **0** stands for the zero vector, and  $N(a, \sigma^2)$  denotes the Gaussian random variable with mean *a* and variance  $\sigma^2$ .

# **2. SYSTEM MODEL**

## 2.1 Received signal

We assume K synchronous users transmit CDMA signals in an additive white Gaussian noise channel. The received continuous signal is sent to a linear input system (preliminary and additional filters [2]) of the multiuser generalized detector whose output is sampled at chip rate. Following the standard chip-sampled discretetime model for a symbol synchronous CDMA system [7], the received vector **x** takes the form

$$\mathbf{x} = \alpha_A (\mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{n}), \tag{1}$$

where

$$\mathbf{x} = [x_1, x_2, \dots, x_N]^T \tag{2}$$

and *N* is the processing gain. The automatic gain control (AGC) scaling the received signal within the dynamic range of the quantizer has a positive gain  $\alpha_A$ .

$$\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_K] \tag{3}$$

is the matrix consisting of 
$$K$$
 spreading sequences where

$$\mathbf{s}_{k} = [s_{k,1}, s_{k,2}, \dots, s_{k,N}]^{T}$$
 (4)  
and

$$s_{k,i} \in \left\{ \frac{1}{\sqrt{N}}, -\frac{1}{\sqrt{N}} \right\}, \quad i = 1, 2, \dots, N .$$
(5)

The diagonal matrix

$$\mathbf{A} = \operatorname{diag}\left(\sqrt{E_1}, \sqrt{E_2}, \dots, \sqrt{E_K}\right)$$
(6)  
is the amplitude matrix with symbol energies  
$$E_1, E_2, \dots, E_K$$

and

$$\mathbf{b} = [b_1, b_2, \dots, b_K]^T, b_k \in \{-1, +1\}$$
(7)

is the transmitted data symbol vector where the elements in  $\mathbf{b}$  are independent to each other. The additive noise vector

$$\mathbf{n} = [n_1, n_2, \dots, n_N]^T \tag{8}$$

is modeled by the normal Gaussian law with  $\mathcal{N}(\mathbf{0}, \sigma_n^2 \mathbf{I})$ ,

where  $\sigma_n^2$  is the noise variance (or noise power).

Assuming the symbol energies  $E_k$  are upper bounded, by the Lindeberg central limit theorem [12], the received CDMA signal  $x_i$ , i = 1, 2, ..., N converges to

$$\mathcal{N}\left(0,\alpha_{\scriptscriptstyle A}^2\left(\frac{1}{N}\sum_{k=1}^K E_k + \sigma_n^2\right)\right)$$

as *K* tends to approach infinity. Without loss of generality, we assume the automatic gain control adjusts the amplitude of received signal  $x_i$  so that the variance of the received signal  $x_i$  is equal to 1 that is to set

$$\alpha_{A}^{2} = \frac{1}{\frac{1}{\frac{1}{N}\sum_{k=1}^{K}E_{k} + \sigma_{n}^{2}}}$$
(9)

Then the input signal  $x_i$  to the quantizer (or analog-todigital converter) can be represented by  $\mathscr{N}(0,1)$  with the following probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp(-0.5x^2), \quad -\infty < x < \infty.$$
 (10)

## 2.2 Uniform quantizer

The operation characteristic of quantizer is given by  $y_k = q(x)$ , if  $x \in (c_k, c_{k+1}]$ , k = 1, 2, ..., L (11) where x is the input signal to the quantizer,  $q(\cdot)$  is the quantization function,

$$L = 2^R \tag{12}$$

is the number of quantization levels and  $R \ge 1$  (13)

is the bit rate of the quantizer,  $y_k$  are the representation levels and  $c_k$  are decision thresholds.

If a quantizer is uniform, the representation levels  $y_k$  and the decision thresholds  $c_k$  can be presented in the following form

$$y_k = \left[k - \frac{L+1}{2}\right]\Delta, \quad k = 2, 3, \dots, L$$
(14)

$$c_k = \left[k - \frac{L+2}{2}\right]\Delta, \quad k = 2, 3, \dots, L \tag{15}$$

where  $\Delta$  is the step size. Moreover, we set

$$c_1 = -\infty$$
, and  $c_{L+1} = \infty$ . (16)

## 3. MULTIUSER GENERALIZED DETECTOR FOR QUANTIZED CDMA SIGNALS IN ADDITIVE WHITE GAUSSIAN NOISE CHANNELS

#### 3.1 Gain plus the additive noise model

To facilitate the design of the multiuser generalized detector based on the generalized approach to signal processing in noise [1]-[5] for the quantized and sampled CDMA signals in the additive white Gaussian noise channel, the output of quantizer after quantization and sampling of the input vector **x** is modeled by using the gain plus the additive noise model [11] which consists of a less than unity gain component  $\alpha_g$  and an additive component  $\zeta_\Sigma$  . We may suggest that the additive component  $\zeta_{\Sigma}$  can be presented in the form of summary uncorrelated interferences: the interference caused by quantization  $\zeta_1$ , normal Gaussian with zero mean and the finite variance  $\sigma_1^2$ , and the interference caused by samp- $\log \zeta_2$  , normal Gaussian with zero mean and the finite variance  $\sigma_2^2$ . Thus, the summary additive interference  $\zeta_{\Sigma}$  can be presented in the form

$$\zeta_{\Sigma} = \zeta_1 + \zeta_2 \tag{17}$$

and is normal Gaussian with the zero mean and finite variance

$$\sigma_{\zeta_{\Sigma}}^2 = \sigma_{\zeta_1}^2 + \sigma_{\zeta_2}^2 \tag{18}$$

This is a direct consequence of the Bussgang's theorem [13].

With this model, the quantizer output vector (after sampling and quantization)

$$\mathbf{z} = [z_1, z_2, \dots, z_N]^T \tag{19}$$

of the input vector **x** takes the following form

$$= \alpha_g \mathbf{x} + \boldsymbol{\zeta}_{\Sigma} = \alpha_g \alpha_A (\mathbf{S}\mathbf{A}\mathbf{b} + \mathbf{n}) + \boldsymbol{\zeta}_{\Sigma}$$
(20)

where the zero mean normal Gaussian quantization and sampling noise vector

$$\boldsymbol{\zeta}_{\Sigma} = [\zeta_{\Sigma_1}, \zeta_{\Sigma_2}, \dots, \zeta_{\Sigma_N}]^T$$
(21)

is uncorrelated with x. Consequently,

$$\boldsymbol{\zeta}_{1} = [\zeta_{1_{1}}, \zeta_{1_{2}}, \dots, \zeta_{1_{N}}]^{T}$$
(22)
and

 $\boldsymbol{\zeta}_{2} = [\zeta_{2_{1}}, \zeta_{2_{2}}, \dots, \zeta_{2_{N}}]^{T}$ are uncorrelated with x, too.

According to the Bussgang's theorem, the parameter  $\alpha_{\varphi}$  is given by

(23)

$$\alpha_{g} = \frac{M[x_{i}z_{i}]}{M[x_{i}x_{i}]} = M[x_{i}z_{i}] = \sum_{k=1}^{L} y_{k} \int_{c_{k}}^{c_{k+1}} xp(x)dx$$
$$= \sum_{k=1}^{L} \frac{y_{k}}{\sqrt{2\pi}} \left(e^{-0.5c_{k}^{2}} - e^{-0.5c_{k+1}^{2}}\right).$$
(24)

The covariance matrix of the additive interference vector  $\zeta_{\Sigma}$  caused by quantization and sampling is determined by  $\sigma_{\zeta_{\Sigma}}^2 \mathbf{I}$  where  $\sigma_{\zeta_{\Sigma}}^2$  can be determined in the following way, too

$$M[\zeta_{\Sigma_{i}}^{2}] = M[\zeta_{\Sigma_{i}}(z_{i} - \alpha_{g}x_{i})] = M[\zeta_{\Sigma_{i}}z_{i}] = M[z_{i}^{2}] - \alpha_{g}^{2}$$
$$= \sum_{k=1}^{L} y_{k}^{2}[Q(c_{k}) - Q(c_{k+1})] - \alpha_{g}^{2}, \qquad (25)$$

where

$$Q(c) = \frac{1}{\sqrt{2\pi}} \int_{c}^{\infty} \exp(-0.5t^2) dt$$
 (26)

is the error integral.

#### 3.2 Multiuser generalized detector

Consider the multiuser generalized detector for quantized and sampled CDMA signals in the additive white Gaussian noise channel based on the generalized approach to signal processing in noise [1]-[5]. Given the sampled and quantized vector z of the process coming in at the input of the multiuser generalized detector for user k, the weight vector of the multiuser generalized detector satisfies the Wiener-Hopf equation [13], which is given by

$$\mathbf{W}_{k} = \mathbf{X}^{-1} \mathbf{a}_{k}, \qquad (27)$$

where

$$\mathbf{X} = M[\mathbf{z} \, \mathbf{z}^T] = M[(\alpha_g \mathbf{x} + \boldsymbol{\zeta}_{\Sigma})(\alpha_g \mathbf{x} + \boldsymbol{\zeta}_{\Sigma})^T]$$
  
=  $(\alpha_g \alpha_A)^2 \mathbf{SAAS}^T + (\alpha_g \alpha_A)^2 \sigma_n^2 \mathbf{I} + \sigma_{\boldsymbol{\zeta}_{\Sigma}}^2 \mathbf{I},$  (28)

$$\mathbf{a}_{k} = M[\mathbf{z} \, b_{k}] = \alpha_{g} \alpha_{A} \mathbf{s}_{k} \sqrt{E_{k}} \,. \tag{29}$$

For user k, the multiuser generalized detector output

$$\mathbf{m}_{k} = [m_{k_{1}}, m_{k_{2}}, \dots, m_{k_{N}}]^{T}$$
(30)

under the main functioning condition of the generalized detector (equality between the parameters of the model signal formed by the receiver, and signal coming in at the input of the generalized detector) takes the following form

$$\mathbf{m}_{k} = 2\mathbf{W}_{k}^{T}\mathbf{z} - \mathbf{z}\,\mathbf{z}^{T} + \mathbf{\eta}_{\Sigma}^{2} = \mathbf{\mu}_{k}\mathbf{b}_{k} + \mathbf{\eta}_{\Sigma}^{2} - \mathbf{\xi}_{\Sigma}^{2}, \qquad (31)$$
  
where

$$\boldsymbol{\mu}_{k} = [\mu_{k_{1}}, \mu_{k_{2}}, \dots, \mu_{k_{N}}]^{T}$$
(32)

represents the amplitude of the signal and

$$\eta_{\Sigma}^2 - \xi_{\Sigma}^2 \tag{33}$$

represents the total background noise and interferences caused by quantization and sampling which is formed at the output of the multiuser generalized detector, where  $\boldsymbol{\xi}_{\Sigma} = \boldsymbol{\xi} + \boldsymbol{\zeta}_{\Sigma} = \boldsymbol{\xi} + \boldsymbol{\zeta}_{1} + \boldsymbol{\zeta}_{2}$ is the noise forming at the output of the preliminary filter of input linear system of the multiuser generalized detector [1]-[5] consisting of the normal Gaussian noise  $\xi$  with the zero mean and the variance  $\sigma_n^2$ , the interference  $\zeta_1$  with the zero mean and the variance  $\sigma_1^2$ , which is caused by quantization, and the interference  $\zeta_2$  with zero mean and the variance  $\sigma_2^2$ , which is caused by sampling. The noise  $\xi$  and the interferences  $\zeta_1$  and  $\zeta_2$  are uncorrelated between each other;

$$\boldsymbol{\eta}_{\Sigma} = \boldsymbol{\eta} + \boldsymbol{\zeta}_{\Sigma} = \boldsymbol{\eta} + \boldsymbol{\zeta}_{1} + \boldsymbol{\zeta}_{2} \tag{35}$$

is the noise forming at the output of the additional filter of input linear system of the multiuser generalized detector [1]-[5] (additional or reference noise) consisting of the normal Gaussian noise  $\eta$  with the zero mean and the variance  $\sigma_n^2$ , the interference  $\zeta_1$  with the zero mean and the variance  $\sigma_1^2$ , which is caused by quantization, and the interference  $\zeta_2$  with zero mean and the variance  $\sigma_2^2$ , which is caused by sampling. The noise  $\eta$  and the interferences  $\zeta_1$  and  $\zeta_2$  are uncorrelated between each other. The probability distribution of the total background noise forming at the output of the multiuser generalized detector is symmetric with respect to 0 because the means of the noise  $\xi$  and  $\eta$  and interferences  $\zeta_1$  and  $\zeta_2$  are equal to zero owing to the initial conditions.

Owing to the fact that the noise  $\xi$  and  $\eta$ , and the interferences  $\zeta_1$  and  $\zeta_2$  are uncorrelated between each other, the variance of the total background noise and interferences at the output of the multiuser generalized detector can be determined in the following form:

$$\sigma_{\eta_{\Sigma}^2 - \zeta_{\Sigma}^2}^2 = 4\alpha_g^4 \alpha_A^4 \sigma_n^4 + 4\sigma_{\zeta_{\Sigma}}^4 .$$
(36)

The variable  $\mu_k$  can be determined in the following form

$$\mu_{k} = M[m_{k}b_{k}] = M[(2\mathbf{W}_{k}^{T}\mathbf{z} - \mathbf{z}\,\mathbf{z}^{T} + \mathbf{\eta}_{\Sigma}^{2})b_{k}] =$$

$$= 2\mathbf{a}_{k}^{T}\mathbf{X}^{-1}\mathbf{a}_{k} - \mathbf{a}_{k}\mathbf{z}^{T} + \mathbf{\eta}_{\Sigma}^{2}$$

$$= E_{k}\mathbf{s}_{k}^{T} \left[\mathbf{SAAS}^{T} + \left(4\sigma_{n}^{4} + \frac{4\sigma_{\zeta_{\Sigma}}^{4}}{\alpha_{g}^{4}\alpha_{A}^{4}}\right)\mathbf{I}\right]^{-1}\mathbf{s}_{k}, \quad (37)$$

The signal-interference ratio (SIR) at the multiuser generalized detector output can be presented in the following form:

$$\delta_{k} = \frac{\mu_{k}^{2}}{\sigma_{\eta_{\Sigma}^{2}-\zeta_{\Sigma}^{2}}^{2}} = \frac{\mu_{k}^{2}}{4(\alpha_{g}^{4}\alpha_{A}^{4}\sigma_{n}^{4} + \sigma_{\zeta_{\Sigma}}^{4})} \quad .$$
(38)

Based on (37), we can define the "equivalent noise variance" of the uniformly quantized CDMA signal in the following form

$$\lambda^{4} = 4 \left( \sigma_{n}^{4} + \frac{\sigma_{\zeta_{\Sigma}}^{4}}{\alpha_{g}^{4} \alpha_{A}^{4}} \right)$$
$$= 4 \sigma_{n}^{4} \left[ 1 + \frac{\sigma_{\zeta_{\Sigma}}^{4}}{\alpha_{g}^{4}} \left( \frac{1}{N^{2}} \sum_{k=1}^{K} \frac{E_{k}^{2}}{\sigma_{n}^{4}} + 1 \right) \right].$$
(39)

From (39), we know the equivalent noise variance is the sum of variances of the background noise forming at the output of the multiuser generalized detector and interferences caused by sampling and quantization and is the function of the parameters  $\alpha_g$  and the variance  $\sigma_{\zeta\Sigma}^4$  of interferences forming at the output of the multiuser generalized detector caused by quantization and sampling of CDMA signals taking into account the gain plus additive noise model. Moreover, the ratio  $\frac{\lambda^4}{\sigma_n^4}$  is an increasing

function of the total signal-to-noise ratio (SNR)  $\sum_{k=1}^{K} \frac{E_k^2}{\sigma_n^4}$ 

by power. That implies the quantization and sampling interferences dominate the bit error rate performance in the high SNR region.

For the unquantized and non-sampled CDMA signals the multiple access interference (MAI) at the multiuser generalized detector output has been shown in [15] to be asymptotically Gaussian. In the case of quantized and sampled CDMA signals, the total background noise and interferences caused by quantization and sampling  $\eta_{\Sigma}^2 - \xi_{\Sigma}^2$  in (31) is also asymptotically Gaussian by the Lindeberg central limit theorem. Since  $b_k$  is binary-phase-shift keying (BPSK) modulated, the bit error rate (BER)  $P_b(k)$  is related to the SIR  $\delta_k$  by the well-known *Q*-function as

$$P_b(k) = Q(\sqrt{\delta_k}) . \tag{40}$$

#### **3.3 Optimum uniform quantizer**

The optimum uniform quantizer in the sense of minimizing the BER performance or maximizing the multiuser generalized detector output SIR is the one maximizing the value of  $\mu_k$ , which is equivalent to minimizing the equivalent noise variance

$$\Delta_{op} = \arg\min_{\Delta} 4 \left( \sigma_n^4 + \frac{\sigma_{\zeta_{\Sigma}}^4}{\alpha_g^4 \alpha_A^4} \right) = \arg\min_{\Delta} \frac{4\sigma_{\zeta_{\Sigma}}^4}{\alpha_g^4}, \quad (41)$$

where we use the fact  $\sigma_n^4$  and  $\alpha_A^4$  are both independent of  $\Delta$ . Substituting (24) and (25) into (41), we obtain

$$\Delta_{op} = \arg \min_{\Delta} \frac{4\sum_{k=1}^{L} y_{k}^{4}(\Delta) [Q(c_{k}(\Delta)) - Q(c_{k+1}(\Delta))]^{2}}{\sum_{k=1}^{L} y_{k}^{2} \left[ e^{c_{k}^{2}(\Delta)} - e^{c_{k+1}^{2}(\Delta)} \right]^{2}},$$
(42)

where we explicitly express  $c_k$  and  $y_k$  as functions of  $\Delta$ . The minimization problem in (42) may be solved by using standard optimization software. For R = 1, the cost function (42) is independent of the step size  $\Delta$  since the quantizer makes hard decision and the multiuser generalized detector becomes a conventional generalized detector. For R = 2,3,...,8, there exists a unique optimal step size  $\Delta_{op}$  for each R. The optimal step sizes  $\Delta_{op}$  and the corresponding parameters  $\alpha_g^4$  and  $\sigma_{\zeta_{\Sigma}}^4$  for R ranging from 1 to 8 are listed in Table 1.

Table 1. Optimum step size for uniform quantizer

R	L	$\Delta_{op}$	$\alpha_g^4$	$\sigma^4_{\zeta_{\Sigma}}$
1	2	0.63158	0.1642	0.0534932
2	4	0.49892	0.6030	0.0109668
3	8	0.38275	0.8586	0.0012986
4	16	0.28948	0.9548	0.0001301
5	32	0.21685	0.9862	0.0000121
6	64	0.16132	0.9960	0.0000011
7	128	0.11927	0.9988	0.0000001
8	256	0.08775	0.9996	0.0000000

#### **4. COMPUTER SIMULATION**

To characterize the effect of sampling and quantization on the multiuser generalized detector, we can rewrite (39) as follows

$$\frac{\lambda^4}{\sigma_n^4} = 4 + \frac{4\sigma_{\zeta_{\Sigma}}^4}{\alpha_g^4} \left(\frac{1}{N^2} \sum_{k=1}^K \frac{E_k^2}{\sigma_n^4} + 1\right) = 4 + \frac{4\sigma_{\zeta_{\Sigma}}^4}{\alpha_g^4} \cdot \bar{\gamma}, \quad (43)$$

where

$$\bar{\gamma} = \frac{1}{N^2} \sum_{k=1}^{K} \frac{E_k^2}{\sigma_n^4} + 1$$
(44)

is an increasing function of K and individual SNR  $\frac{E_k^2}{4}$ .

The parameter  $\overline{\gamma}$  is the variance of the received CDMA signal before the automatic gain control divided by the squared additive white Gaussian noise variance  $\sigma_n^4$ .

Figure 1 shows the relationship between  $\frac{\lambda^4}{\sigma_{\pi}^4}$  and  $\overline{\gamma}$  for

a number of quantization levels. From Fig. 1, we learn the equivalent noise variance caused by quantization and sampling increases as the parameter  $\overline{\gamma}$  becomes larger or the number of bit rate *R* becomes smaller. For example, when  $\overline{\gamma}$  is equal to 20 dB, data quantization and sampling increases the noise variance by 0.2 dB, 0.7 dB, 1.85 dB, and 4.1 dB for *R* = 4,3,2,1, respectively.

The bit error rate (BER) performance of the multiuser generalized detector for the optimally quantized and sampled, random spreading CDMA signals obtained by computer simulation are shown in Fig. 2, where N = 64 and K = 32, and each user has equal received power. The computer simulation has been carried out in the following way. First, we generate and uniformly quantize and sample the received CDMA signal according to (1) and (11), respectively. Then the quantized and sampled

signal is filtered by the input linear tract of the multiuser generalized detector and processed using the weight vector determined based on (27). Finally, we detect the transmitted bit  $b_k$  from the multiuser generalized detector output, which is given in (31).



Fig.1. The normalized equivalent noise variance  $\frac{\lambda^4}{\sigma_n^4}$  versus the sum of SNRs.

Also, the BER performance for MMSE multiuser receiver discussed in [6] is presented in Fig.2. Comparative analysis of the presented curves allows us to make a conclusion that the multiuser generalized detector has a great superiority in comparison with the MMSE multiuser receiver [6]. For example, at  $BER = 10^{-4}$ , the SNR loss due to quantization and sampling for MMSE multiuser receiver from [6] is about 3.6 dB and 1.2 dB for R = 3 and R = 4, respectively. In the case of the multiuser generalized detector, the SNR loss due to quantization and sampling at  $BER = 10^{-4}$  is about 1.3 dB and 0.2 dB for R = 3 and R = 4, respectively.

#### **5. CONCLUSIONS**

From analysis of computer simulation results we see that the equivalent noise variance caused by quantization and sampling is a function of the parameter  $\overline{\gamma}$  or a function of the number of bit rate *R*. The equivalent noise variance caused by quantization and sampling increases as the parameter  $\overline{\gamma}$  becomes larger or the number of bit rate *R* becomes smaller.

BER performance of the multiuser generalized detector surpasses the same performance of MMSE receiver discussed in [6]. We see that at the same value of SNR, firstly, the BER performance for the multiuser generalized detector is much lower in comparison with the BER performance for MMSE receiver [6], and, secondly, we observe a superiority in SNR loss caused by quantization and sampling under the use of the multiuser generalized detector in comparison with employment of MMSE



Fig.2. BER for the uniformly quantized and sampled CDMA signal with multiuser generalized detector and perfect control. N = 64, K = 32.

receiver. Thus, at  $BER = 10^{-4}$ , the SNR loss due to quantization and sampling for MMSE multiuser receiver [6] is about 3.6 dB and 1.2 dB for R = 3 and R = 4, respectively. In the case of the multiuser generalized detector, the SNR loss at the same  $BER = 10^{-4}$  is about 1.3 dB and 0.2 dB for R = 3 and R = 4, respectively. The use of the multiuser generalized detector constructed on the basis of the generalized approach to signal processing in noise [1]–[5] has very good perspectives under employment of CDMA signals.

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