Optimal Combining, Partial Cancellation, and Channel Estimation and Correlation in DS-CDMA Systems Employing the Generalized Detector

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Abstract: - The symbol-error rate (SER) of a quadrature subbranch hybrid selection/maximal-ratio combining scheme for 1-D modulations in Rayleigh fading under employment of the generalized detector (GD), which is constructed based on the generalized approach to signal processing in noise, is investigated. At the GD, N diversity branches are split into $2N$ in-phase and quadrature subbranches. Traditional hybrid selection/maximal-ratio combining is then applied over $2N$ subbranches. M-ary pulse amplitude modulation, including coherent bi-nary phase-shift keying (BPSK), with quadrature subbranch hybrid selection/maximal-ratio combining is investigated. The SER performance of the GD under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes are investigated and compared with the conventional hybrid selection/maximal-ratio combining receivers. The obtained results show that the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes outperform the traditional hybrid selection/maximal-ratio combining receiver. Procedure of selecting the partial cancellation factor for the first stage of a hard-decision partial parallel interference cancellation of the GD employing in direct-sequence code-division multiple access (DS-CDMA) systems is proposed. A range of the optimal partial cancellation factor, where the lower and upper boundary values can be explicitly calculated from the processing gain and the number of users of the DS-CDMA system, is derived based on the Price's theorem. Computer simulation results confirmed that, using the average of these two boundary values as the partial cancellation factor for the first stage, we are able to reach the bit error rate (BER) performance that is very close to the potentially achieved BER performance using the GD and surpasses the BER performance of the real partial cancellation factor for DS-CDMA systems discussed in literature. Channel estimation errors have to be taken into account under analysis of receiver in order to the system performance will be not degraded under practical channel estimation. In this paper, we propose the GD, which takes the estimation error of maximum likelihood (ML) multiple-input multiple-output (MIMO) channel estimation and receiver spatially correlation into account in computation of the minimum mean square error (MMSE) GD and log-likelihood ratio (LLR) of each coded bit. Comparative analysis and simulation results show that the proposed MMSE GD can obtain sizable performance gain and outperforms the existing soft-output MMSE vertical Bell Lab Space Time (V-BLAST) detector and its modification versions.

Key Words: - Generalized detector, Diversity combining, Symbol error rate, Fading channel, Hybrid selection/ maximal-ratio combining, Partial cancellation factor, Direct-sequence code-division multiple access (DS-CDMA), Partial parallel interference cancellation (PPIC), Channel estimation, Channel correlation, MMSE, V-BLAST detector.

1 Introduction

In this paper we investigate the generalized detector (GD), which is constructed based on the generalized approach to signal processing (GASP) in noise [1]–[5], under quadrature subbranch hybrid selection/maximal-ratio combining for 1-D modulations in Rayleigh fading and compare its symbol error rate (SER) performance with that of the traditional hybrid selection/maximal-ratio combining scheme discussed in [6]. Also, we investigate the problem of selecting the partial cancellation factor for the first stage of a hard-decision partial parallel interference cancellation of the GD employed by direct-sequence code-division multiple access (DS-CDMA) system. Additionally we take into account channel estimation errors under analysis of the GD in order to the DS-CDMA system performance will be not degraded at practical channel estimation.

It is well known that the hybrid selection/maximal-ratio combining receiver selects the L strongest signals from N available diversity branches and coherently combines them [7]–[13]. In traditional hybrid selection/maximal-ratio combining scheme, the strongest L signals are selected according to signal-envelope amplitude [7]–[13]. However, some receiver implementations recover directly the in-phase (I) and quadrature (Q) components of the received branch signals Furthermore, optimal maximum likelihood estimation of the phase of a diversity branch signal is implemented by first estimating the in-phase and quadrature branch signal components and obtaining the signal phase as a derived quantity [14] and [15]. Other channel estimation procedures also operate by first estimating the in-phase and quadrature branch signal components [16]–[18]. Thus, rather than N available signals, there are $2N$ available quadrature branch signal components for combining.

In general, the largest $2L$ of these $2N$ quadrature branch signal components will not be the same as the $2L$ quadrature branch signal components of the L branch signals having the largest signal envelopes.

In this paper, we investigate how much improvement in performance can be achieved by using the GD constructed in accordance with GASP [1]–[5] with modified hybrid selection/maximal-ratio combining, namely, with quadrature subbranch hybrid selection/ maximal-ratio combining and hybrid selection/maximal-ratio combining schemes, instead of the conventional hybrid selection/maximal-ratio combining scheme for 1-D signal modulations in Rayleigh fading.

At the GD, the N diversity branches are split into $2N$ in-phase and quadrature subbranches. Then the GD with hybrid selection/maximal-ratio combining scheme [19] is applied to these $2N$ subbranches. Obtained results show that the better performance is achieved by this quadrature subbranch hybrid selection /maximal-ratio combining scheme in comparison with the traditional hybrid selection/maximal-ratio combining scheme for the same value of average signal-to-noise ratio (SNR) per diversity branch.

Another problem discussed is the problem of partial interference cancellation. It is well known that the multiple access interference can be efficiently estimated by the partial parallel interference cancellation procedure and then partially be cancelled out of the received signal on a stage-by-stage basis for DS-CDMA system [20].

To ensure good performance, the partial cancellation factor for each partial parallel interference cancellation stage needs to be chosen appropriately, where the partial cancellation factor should be increased as the reliability of the multiple access interference estimates improves. There are some papers on the selection of the partial cancellation factors for a detector based on the partial parallel interference cancellation. In [21]–[23], formulas of the optimal partial cancellation factors were derived through straightforward analysis based on soft decisions. In contrast, it is very difficult to obtain the optimal partial cancellation factors for a detector based on the partial parallel interference cancellation with hard decisions owing to their nonlinear character.

One common approach to solve the nonlinear problem is to select an arbitrary partial cancellation factor for the first stage and then increase the value for each successive stage, since the multiple access interference estimates may become more reliable in later stages [20], [24], [25]. This approach is simple, but it might not provide satisfactory performance.

In this paper, we use Price's theorem [26], [27] to derive a range of the optimal partial cancellation factor for the first stage in an additive white Gaussian noise environment employing the GD designed based on GASP [1]–[5], where the lower and upper boundary values of the partial cancellation factor can be explicitly calculated from the processing gain and the number of users of the DS-CDMA.

Computer simulation results show that, using the average of these two boundary values as the partial cancellation factor for the first stage, we are able to reach the bit error rate performance (BER) that is very close to the potentially achieved BER under the use of the GD [28] and surpasses the BER of the real partial cancellation factor for DS-CDMA systems discussed in [29].

With this result, a reasonable partial cancellation factor can quickly be determined without the use of time-consuming Monte Carlo simulations. It is worth mentioning that the two-stage GD [30] based on the partial parallel interference cancellation using the proposed partial cancellation factor at the first stage achieves the BER performance comparable to that of the three-stage GD based on the partial parallel interference cancellation using an arbitrary partial cancellation factor at the first stage.

In other words, under the same BER performance, the proposed approach for selecting the partial cancellation factor can reduce the complexity of the GD based on the partial parallel interference cancellation. Although the partial cancellation factor selection approach is derived for an additive white Gaussian noise environment under employing the GD constructed on the basis of GASP [1]–[5], it can be applied to multipath fading cases [31], [32].

In channel estimation and correlation part, we consider the minimum mean square error (MMSE) GD constructed based on the GASP [1]–[5], which takes the estimation error of maximum likelihood (ML) multiple-input multiple-output (MIMO) channel estimation and receiver spatially correlation into account in the computation of MMSE GD and log-likelihood ratio (LLR) of each coded bit. Investigation and analysis needs of the GD are based on the following statements.

The existing soft-output MMSE vertical Bell Lab Space Time (V-BLAST) detectors are obtained based on perfect channel estimation [33]–[35]. Unfortunately, the estimated MIMO channel coefficients matrix is noisy and imperfect in practical application environments [36], [37]. Therefore, these soft-output MMSE V-BLAST detectors will suffer from performance degradation under practical channel estimation.

In [37] the ML symbol detection scheme, which takes into consideration the channel estimation error was investigated when the MIMO channel estimation is imperfect. Recently, a MMSE based V-BLAST symbol detection algorithm, which addresses the impact of the channel estimation error, was discussed in [38]. However, no channel coding, decision error propagation compensation and spatially channel correlation are considered in [38]. There was made an attempt in [39] to overcome the shortcomings of the existing soft-output MMSE V-BLAST detectors. The soft-output MMSE V-BLAST detector that takes the estimation error of ML MIMO channel estimation and receiver spatially correlation into account was proposed in [39].

In the present paper, we derive the MMSE GD for detecting each transmitted symbol and provide the method to compute the LLR of each coded bit. When compared with the latest modified detection scheme version of soft-output MMSE V-BLAST detector discussed in [39], our simulation results show that the MMSE GD can obtain sizable performance gain.

The organization of this paper is as follows. Section II presents the system model for quadrature subbranch hybrid selection/maximal-ratio combining and selection/maximal-ratio combining schemes, synchronous DS-CDMA system, and MIMO system. Section III describes the performance analysis for quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes, in which we obtain the SER expression and define the moment generating function of SNR per symbol at the output of the GD. Section IV is devoted to determination of the partial cancellation factor for the first stage of the partial parallel interference cancellation. Section V deals with a channel estimation and computation of MMSE GD and LLR; also there are some remarks concerning to MMSE GD. In Section VI, we compare SER performance for the GD under quadrature subbranch hybrid selection/maximal-ratio combining with the traditional combining receiver, the BER performance of the single stage GD is compared with that of conventional receiver discussed in [29], and the BER performance of different GDs under spatially independent MIMO channel and receiver spatially correlated MIMO channel. Finally, the conclusions are given in Section VII.

2 Problem Statement and System Model

2.1 Selection/Maximal-Ratio Combining

We assume that there are N diversity branches experiencing slow and flat Rayleigh fading, and all of the fading processes are independent and identically distributed. During analysis we consider only the hypothesis H_1 "a yes" signal in the input stochastic process. Then the equivalent received baseband signal for the k-th diversity branch takes the following form:

$$x_k(t) = h_k a(t) + n_k(t)\,, \quad k = 1, \ldots, N \ , \tag{1}$$

where $a(t)$ is a 1-D baseband transmitted signal that, without loss of generality, is assumed to be real, h_k is the channel gain for the k-th branch subjected to Rayleigh fading, and $n_k(t)$ is a zero-mean white complex Gaussian noise process with two-sided power spectral density $\frac{N_0}{2}$ with the dimension $\frac{W}{Hz}$.

At the GD front end, for each diversity branch, the received signal is split into its in-phase and quadrature signal components. Then, the conventional hybrid selection/maximal-ratio combining scheme is applied over all of these quadrature branches, as shown in Fig.1.

We can present h_k as

$$h_k = h_{kI} + jh_{kQ} \tag{2}$$

and $n_k(t)$ as

$$n_k(t) = n_{kI}(t) + jn_{kQ}(t) \ , \tag{3}$$

the in-phase signal component $x_{kI}(t)$ and quadrature

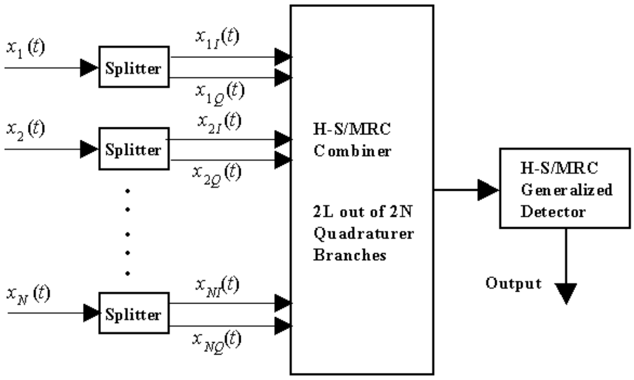

Figure 1. Block diagram of the generalized receiver under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes.

signal component $x_{kQ}(t)$ of the received signal $x_k(t)$ are given by

$$x_{kI}(t) = h_{kI}a(t) + n_{kI}(t) \quad , \tag{4}$$

$$x_{kQ}(t) = h_{kQ}a(t) + n_{kQ}(t) \quad . \tag{5}$$

Since h_k $(k = 1,\dots,N)$ are subjected to independent and identically distributed Rayleigh fading, h_{kI} and h_{kQ} are independent zero-mean Gaussian random variables with the same variance [27]

$$D_h = \frac{1}{2}M\{|h_k|^2\} \quad . \tag{6}$$

Further, the in-phase $n_{kI}(t)$ and quadrature $n_{kQ}(t)$ noise components are also independent zero-mean Gaussian random processes, each with two-sided power spectral density $\frac{N_0}{2}$ with the dimension $\frac{W}{Hz}$ [14]. Due to the independence of in-phase h_{kI} and quadrature h_{kQ} channel gain components and in-phase $n_{kI}(t)$ and quadrature $n_{kQ}(t)$ noise components, the $2N$ quadrature branch received signal components conditioned on the transmitted signal are independent and identically distributed. We can reorganize the in -phase and quadrature components of the channel gains h_k and Gaussian noise $n_k(k = 1,\dots,N)$ as g_k and $\xi_k(t)$, given, respectively, by

$$g_k = \begin{cases} h_{kI}\,, & k = 1,\dots,N \\ h_{(k-N)Q}\,, & k = N+1,\dots,2N\,; \end{cases} \tag{7}$$

$$\xi_k(t) = \begin{cases} \xi_{kI}(t)\,, & k = 1,\dots,N \\ \xi_{(k-N)Q}(t)\,, & k = N+1,\dots,2N\,. \end{cases} \tag{8}$$

The signal at the output of the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes takes the following form:

$$Z^{g}_{QBHS/MRC}(t) = s^2(t)\sum_{k=1}^{2N} c_k^2 g_k^2 + \sum_{k=1}^{2N} c_k^2 g_k^2[\eta_k^2(t) - \xi_k^2(t)] \tag{9}$$

where $\eta_k^2(t) - \xi_k^2(t)$ is the background noise [5] forming at the output of the generalized detector for the k-th branch;

$$\eta_k(t) = \begin{cases} \eta_{kI}(t)\,, & k = 1,\dots,N \\ \eta_{(k-N)Q}(t)\,, & k = N+1,\dots,2N\,; \end{cases} \tag{10}$$

$\eta_k(t)$ is the reference zero-mean white complex Gaussian noise process with two-sided power spectral density $\frac{N_0}{2}$ with the dimension $\frac{W}{Hz}$ introduced according to GASP [2]; $c_k \in \{0,1\}$ and $2L$ of c_k equal to 1.

2.2 Synchronous DS-CDMA System

Consider a synchronous DS-CDMA system employing the GD with K users and a processing gain N. The following form can present the received signal:

$$x(t) = \sum_{k=1}^{K} S_k h_k b_k c_k(t) + n(t) = \sum_{k=1}^{K} S_k h_k b_k \sum_{i=1}^{N} c_{ki} a(t - iT_c) + n(t), \quad t \in [0, T_b] \tag{11}$$

where S_k is the received signal amplitude for the k-th user, b_k represents the transmitted bit taking value on ± 1 equiprobably, $\{c_{k1}, c_{k2},..., c_{kN}\}$ is a random spreading code with each element taking value on $\frac{\pm 1}{\sqrt{N}}$ equiprobably, $a(t)$ is a normalized chip waveform of duration T_c,

$$c_k(t) = \sum_{i=1}^{N} c_{ki} a(t - iT_c) \tag{12}$$

is the signature waveform of the k-th user, $n(t)$ is the zero mean additive white Gaussian noise with the power spectrum density $0.5N_0$, as before, and $T_b = NT_c$ is the bit duration.

Using the GD based on the multistage partial parallel interference cancellation for the DS-CDMA system and assuming that the user l is the desired user, we can express the corresponding output of the GD in according to GASP as follows:

$$Z_l = \int_0^{T_b} 2x(t)c_l(t)dt - \int_0^{T_b} x(t)x(t-\tau)dt \tag{13}$$

where τ is the delay factor that can be neglected for simplicity of analysis. For this case, we have

$$Z_l = S_l h_l b_l + \sum_{k \neq l}^{K} S_k h_k b_k \rho_{kl} + \zeta_l = S_l h_l b_l + I_l + \zeta_l \tag{14}$$

where

$$\rho_{kl} = \int_0^{T_b} c_k(t)c_l(t)dt \tag{15}$$

is the coefficient of correlation between signature waveforms of the k-th and l-th users;

$$\zeta_l = \int_0^{T_b} [\eta^2(t) - \xi^2(t)]c_l(t) \tag{16}$$

is the total noise component at the output of the GD, where $\eta^2(t) - \xi^2(t)$ is the background noise forming at the output of the universally adopted GD [5] – here $\xi(t)$ is the noise forming at the output of the preliminary filter (PF) of input linear tract of the GD, $\eta(t)$ is the noise forming at the output of the additional filter (AF) of input linear tract of the GD; the noise $\xi(t)$ and $\eta(t)$ are uncorrelated between each other owing to choice of amplitude-frequency responses and parameters of the PF and AF and have the same statistical characteristics as the noise $n(t)$ at the input of the GD [2];

$$I_l = \sum_{k \neq l}^{K} S_k h_k b_k \rho_{kl} \tag{17}$$

is the multiple access interference.

Denoting the soft and hard decisions of the GD output for user l by

$$\tilde{b}_l^{(0)} = Z_l \quad \text{and} \quad \hat{b}_l^{(0)} = sgn(Z_l) \tag{18}$$

respectively, the output of the first partial parallel interference cancellation stage with a partial cancellation factor equal to a_1 can be written by [20]

$$\tilde{b}_l^{(1)} = a_1(Z_l - \hat{I}_l) + (1-a_1)\tilde{b}_l^{(0)} = Z_l - a_1\hat{I}_l \tag{19}$$

where $\tilde{b}_l^{(1)}$ denotes the soft decision of the user l at the first stage of partial parallel interference cancellation and

$$\hat{I}_l = \sum_{k \neq l}^{K} S_k \hat{b}_k^{(0)} \rho_{kl} \tag{20}$$

is the estimated multiple access interference using hard decision.

2.3 MIMO System

In a MIMO system with N_T transmitter antennas and N_R receiver antennas, the received signal can be expressed in the following form:

$$\mathbf{y} = \mathbf{Ha} + \mathbf{n} = \sum_{k=1}^{N_T} \mathbf{h}_k a_k + \mathbf{n} \ , \tag{21}$$

where

$$\mathbf{y} = [y_1, y_2, \ldots, y_{N_R}]^T \tag{22}$$

is the received signal vector;

$$\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{N_T}]^T \tag{23}$$

is the $N_R \times N_T$ MIMO channel coefficients matrix with elements $h_{k,l}$ denoting the channel fading coefficient between the k-th transmitter antenna and the l-th receiver antenna.

In this paper, we adopt the following receiver spatially correlated MIMO channel model

$$\mathbf{H} = \sqrt{\mathbf{R_r}}\,\mathbf{H}_w \tag{24}$$

with $\mathbf{H}_w$ denoting an independent and identically distributed matrix with entries obeying Gaussian law with zero mean and unit variance, and $\mathbf{R_r}$ is $N_R \times N_R$ receive array correlation matrix determined by

$$\mathbf{R_r} = \sqrt{\mathbf{R_r}}(\sqrt{\mathbf{R_r}})^H \ . \tag{25}$$

Then, we have

$$M\{\mathbf{HH}^H\} = N_T \mathbf{R_r} \ , \tag{26}$$

where $M\{\cdot\}$ is the mathematical expectation. The channel is considered to be flat fading with coherence time of $(N_P + N_D)$ MIMO vector symbols, where N_P symbol intervals are dedicated to pilot matrix $\mathbf{S}_p$ and the remaining N_D to data transmission.

$$\mathbf{a} = [a_1, a_2, \ldots, a_{N_T}]^T \tag{27}$$

is the transmitted complex signal vector whose element is taken from a complex modulation constellation $\mathscr{F}$, such as quadriphase shift keying (QPSK) signal, by mapping the coded bit vector

$$\mathbf{g}_k = [\mathbf{g}_k^1, \mathbf{g}_k^2, \ldots, \mathbf{g}_k^{\log_2 M}]^T \tag{28}$$

to one modulation symbol belonging to $\mathscr{F}$, i.e.,

$$a_k = \mathrm{map}(\mathbf{g}_k) \in \mathscr{F} \ . \tag{29}$$

Meanwhile, we assume that each transmitted symbol is independently taken from the same modulation constellation and has the same average energy, i.e.,

$$M\{\mathbf{aa}^H\} = E_a \mathbf{I}_{N_T} \ . \tag{30}$$

Finally,

$$\mathbf{n} = [n_1, n_2, \ldots, n_{N_R}]^T \tag{31}$$

is the additive white Gaussian noise (AWGN) vector with covariance matrix determined by

$$M\{\mathbf{nn}^H\} = \sigma_n^2 \mathbf{I}_{N_R} \ . \qquad (32)$$

$\mathbf{I}_{N_T}$ and $\mathbf{I}_{N_R}$ are the identity matrices.

3 Performance Analysis

3.1 Symbol Error Rate Expression

Let q_k denote the instantaneous signal-to-noise ratio per symbol of the k-th quadrature branch ($k = 1,\ldots, 2N$) at the output of the GD under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes. In line with [2], this instantaneous (SNR) q_k can be defined as

$$q_k = \frac{E_b}{\sqrt{4\sigma_n^4}} g_k^2 \ , \qquad (33)$$

where E_b is the average symbol energy of the transmitted signal $a(t)$. Assume that we choose $2L$ ($1 \leq L \leq N$) quadrature branched out of the $2N$ branches. Then, the SNR per symbol at the GD output at quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes may be presented as

$$q_{QBHS/MRC} = \sum_{k=1}^{2L} q_{(k)} \ , \qquad (34)$$

where $q_{(k)}$ are the ordered instantaneous SNRs q_k and satisfy the following condition

$$q_{(1)} \geq q_{(2)} \geq \cdots \geq q_{(2N)} \ . \qquad (35)$$

When $L = N$, we obtain the maximal-ratio combiner, as expected.

Using the moment generating function method discussed in [11] and [40], the SER of an M-ary pulse amplitude modulation (PAM) system conditioned on $q_{QBHS/MRC}$ is given by

$$P_s(q_{QBHS/MRC}) = \frac{2(M-1)}{M\pi} \times \int_0^{0.5\pi} \exp\Big(-\frac{g_{M-PAM}}{\sin^2\theta} q_{QBHS/MRC}\Big) d\theta \ , \qquad (36)$$

where

$$g_{M-PAM} = \frac{3}{M^2 - 1} \ . \qquad (37)$$

Averaging (36) over $q_{QBHS/MRC}$, the SER of the M-ary pulse amplitude modulation system is determined in the following form:

$$P_s = \frac{2(M-1)}{M\pi} \times \int_0^{0.5\pi}\int_0^{\infty} \exp\Big(-\frac{g_{M-PAM}}{\sin^2\theta} q\Big) f_{q_{QBHS/MRC}}(q)\,dq\,d\theta = \frac{2(M-1)}{M\pi}\int_0^{0.5\pi} \varphi_{q_{QBHS/MRC}}\Big(-\frac{g_{M-PAM}}{\sin^2\theta}\Big) d\theta \ , \qquad (38)$$

where

$$\varphi_q(s) = M_q\{\exp(sq)\} \qquad (39)$$

is the moment generating function of random variable q, $M_q\{\cdot\}$ is the mathematical expectation of the moment generating function with respect to SNR per symbol.

When $M = 2$, the average BER performance of a coherent binary phase-shift keying (BPSK) system using the quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes can be determined in the following form:

$$P_b = \frac{1}{\pi}\int_0^{0.5\pi} \varphi_{q_{QBHS/MRC}}\Big(-\frac{1}{\sin^2\theta}\Big) d\theta \ . \qquad (40)$$

3.2 Moment Generating Function of $q_{QBHS/MRC}$

Since all of the $2N$ quadrature branches are independent and identically distributed, the moment generating function of $q_{QBHS/MRC}$ takes the following form [13]:

$$\varphi_{q_{QBHS/MRC}}(s) = 2L\binom{2N}{2L} \times \int_0^{0.5\pi} \exp(sq) f(q)[\varphi(s,q)]^{2L-1}[F(q)]^{2(N-L)} dq \ , \qquad (41)$$

where $f(q)$ and $F(q)$ are, respectively, the probability density function and the cumulative distribution function of q, the SNR per symbol, for each quadrature branch, and

$$\varphi(s,q) = \int_q^{\infty} \exp(sx) f(x) dx \qquad (42)$$

is the marginal moment generating function of the SNR per symbol of a single quadrature branch.

Since g_k and g_{k+N} ($k = 1,\ldots, N$) follow a zero-mean Gaussian distribution with the variance D_h given by (6), one can show that q_k and q_{k+N} follow the Gamma distribution with probability density function given by [27]

$$f(q) = \begin{cases} \dfrac{1}{\sqrt{q}} \exp(-\dfrac{q}{\beta})\sqrt{\pi \overline{q}}\,, & q \ge 0 \\ 0\,, & q \le 0\,, \end{cases} \quad (43)$$

where

$$\overline{q} = \frac{2E_b D_h}{\sqrt{4\sigma_n^4}} \quad (44)$$

is the average SNR per symbol for each diversity branch.

Then the marginal moment generating function of the SNR per symbol of a single quadrature branch can be determined in the following form:

$$\varphi(s,q) = \frac{1}{\sqrt{1-s\overline{q}}}\, erfc\left(\sqrt{\frac{1-s\overline{q}}{\overline{q}}q}\right) \quad (45)$$

and the cumulative distribution function of q becomes

$$F(q) = 1 - \varphi(0,q) = 1 - erfc\left(\sqrt{\frac{q}{\overline{q}}}\right)\,, \quad (46)$$

where

$$erfc\,(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} \exp(-t^2)dt \quad (47)$$

is the error function.

4 Determination of the Partial Cancellation Factor

In this section we determine the partial cancellation factor for the first stage of the partial parallel interference cancellation. From [20] and [29], the linear MMSE solution of the partial cancellation factor for the first stage of the partial parallel interference cancellation is given by

$$a_{1,\text{opt}} = \frac{\sigma_{2,0}^2 - \rho_1\sigma_{1,1}\sigma_{2,0}}{\sigma_{1,1}^2 + \sigma_{2,0}^2 - 2\rho_1\sigma_{1,1}\sigma_{2,0}} \quad (48)$$

where

$$\sigma_{1,1}^2 = M\{(I_l + \zeta_l - \hat{I}_l)^2\} \quad (49)$$

is the power of the residual multiple access interference plus the background noise forming at the output of the GD at the first stage, $M\{\cdot\}$ denotes the mean;

$$\sigma_{2,0}^2 = M\{(I_l + \zeta_l)^2\} \quad (50)$$

is the power of the true multiple access interference plus the background noise forming at the output of the GD (also called the 0-th stage);

$$\rho_1\sigma_{1,1}\sigma_{2,0} = M\{(I_l + \zeta_l - \hat{I}_l)(I_l + \zeta_l)\} \quad (51)$$

is the correlation between these two interference terms. It can be rewritten as

$$a_{1,\text{opt}} = \frac{M\{(I_l + \zeta_l)\hat{I}_l\}}{M\{\hat{I}_l^2\}}$$

$$= \frac{1}{\frac{1}{N}\sum\limits_{u\ne l}^{K} S_u^2 + \sum\limits_{u\ne l}^{K}\sum\limits_{v\ne l,u}^{K} S_u S_v M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}\hat{b}_v^{(0)}\}} \times$$

$$\Big\{\frac{1}{N}\sum_{u\ne l}^{K} A_u^2(1-2P_{e,u}) + \sum_{u\ne l}^{K}\sum_{v\ne l,u}^{K} S_u S_v M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}\hat{b}_v^{(0)}\}$$

$$+ \sum_{v\ne l}^{K} S_v M\{\rho_{vl}\zeta_l \hat{b}_v^{(0)}\}\Big\} \quad (52)$$

where $P_{e,u}$ is the BER of user u at the corresponding output of the GD;

$$M\{\hat{b}_u^{(0)}\hat{b}_u^{(0)}\} = 1 - 2P_{e,u} \quad \text{and} \quad M\{\rho_{ul}^2\} = \frac{1}{N}\,. \quad (53)$$

We drop here the channel gain h_k. We analyze this factor in detail in next Section.

The partial cancellation factor $a_{1,\text{opt}}$ can be regarded as the normalized correlation between the true multiple access interference plus the background noise forming at the output of the GD and the estimated multiple access interference.

Assume that

$$\mathbf{b} = \{b_k\}_{k=1}^{K} \quad (54)$$

is the data set of all users;

$$\boldsymbol{\rho} = \{\rho_{kl}\}_{k,l=1}^{K} \quad (55)$$

is the correlation coefficient set of random sequences

$$f_{\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(\tilde{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho}) = N(M\{\tilde{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\}, \sigma^2) \quad (56)$$

is the conditional normal probability density function of $\tilde{b}_v^{(0)}$ given $\mathbf{b}$ and $\boldsymbol{\rho}$ and $f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(\tilde{b}_u^{(0)},\tilde{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho})$ is the conditional joint normal probability density function of $\tilde{b}_u^{(0)}$ and $\tilde{b}_v^{(0)}$ given $\mathbf{b}$ and $\boldsymbol{\rho}$. Following the derivations in [20] and [29], the expectation terms with hard decisions in (52) can be evaluated based on Price's theorem [27] as follows

$$M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}\hat{b}_v^{(0)}\}$$

$$= M\{M\{M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}\hat{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\} \mid \boldsymbol{\rho}\}\}$$

$$= M\{M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}(2Q_v - 1) \mid \boldsymbol{\rho}\}\} \quad (57)$$

$$M\{\rho_{vl}\zeta_l\hat{b}_v^{(0)}\} = M\{M\{M\{\rho_{vl}\zeta_l\hat{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\} \mid \boldsymbol{\rho}\}\}$$

$$= 2\sigma^2 M\{M\{\rho_{vl}^2 f_{\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0 \mid \mathbf{b},\boldsymbol{\rho}) \mid \boldsymbol{\rho}\}\} \quad (58)$$

$$M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}\hat{b}_v^{(0)}\}$$
$$= M\{M\{M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}\hat{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\} \mid \boldsymbol{\rho}\}\}$$
$$= M\{M\{\rho_{ul}\rho_{vl}[4\rho_{uv}\sigma^2 f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho})$$
$$+ (2Q_u - 1)(2Q_v - 1)] \mid \boldsymbol{\rho}\}\} \qquad (59)$$

where

$$Q_l = Q\Big(-\frac{M\{\tilde{b}_l^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\}}{\sigma}\Big) \qquad (60)$$

and

$$\sigma^2 = Var\{\zeta_l\} = 4\sigma_n^4 \qquad (61)$$

is the variance of the total background noise forming at the output of the GD; σ_n^2 is the variance of the additive Gaussian noise at the outputs of the PF and AF of linear tract of the GD;

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp(-0.5t^2)dt \qquad (62)$$

is the probability integral.

Although numerical integration in [20], [22], and [29] can be adopted for determining the optimal partial cancellation factor $a_{1,\text{opt}}$ for the first stage based on (52)-(59), it requires huge computational complexity. To simplify this problem, we assume that the total background noise forming at the GD output can be considered as a constant factor and may be small enough such that the Q functions in (57) and (59) are all constants and (58) can be approximated to zero. That is

$$\sigma^2 << \min_{\{S_k,\boldsymbol{\rho}\}}(M\{\tilde{b}_u^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\})^2 = \frac{4S_m^2}{N^2} \qquad (63)$$

where [41]

$$\begin{cases} S_m = \min S_k \\ \sum_{k \neq m}^{K} S_k b_k \rho_{kl} = -S_m b_m \rho'_{ml} \\ \min|\rho_{ml} - \rho'_{ml}| = \frac{2}{N} \end{cases} \qquad (64)$$

With this, we can rewrite (57) and (59) as follows:

$$M\{M\{\rho_{ul}\rho_{vl}\hat{b}_u^{(0)}(2Q_v - 1) \mid \boldsymbol{\rho}\}\}$$
$$= B_1 M\{\rho_{ul}\rho_{vl}\}M\{\hat{b}_u^{(0)} \mid \boldsymbol{\rho}\} = 0 \qquad (65)$$
$$M\{M\{\rho_{ul}\rho_{vl}[4\rho_{uv}\sigma^2 f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho})$$
$$+ (2Q_u - 1)(2Q_v - 1)] \mid \boldsymbol{\rho}\}\}$$
$$= M\{M\{4\sigma^2\rho_{ul}\rho_{vl}\rho_{uv} f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho}) \mid \boldsymbol{\rho}\}\}$$
$$+ B_2 M\{M\{\rho_{ul}\rho_{vl} \mid \boldsymbol{\rho}\}\}$$
$$= M\{M\{4\sigma^2\rho_{ul}\rho_{vl}\rho_{uv} f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho}) \mid \boldsymbol{\rho}\}\} \qquad (66)$$

where B_1 and B_2 are constants. According to the assumptions made above, $f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho})$ can be expressed by

$$f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho}) = \frac{\exp(-\frac{1}{2}\mathbf{m}_b^T\mathbf{B}_b^{-1}\mathbf{m}_b)}{2\pi\sigma^2\sqrt{1-\rho_{uv}^2}} \qquad (67)$$

where

$$\mathbf{m}_b = [M\{\tilde{b}_u^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\}, M\{\tilde{b}_v^{(0)} \mid \mathbf{b},\boldsymbol{\rho}\}]^T \qquad (68)$$

and

$$\mathbf{B}_b = M\{(\tilde{\mathbf{b}} - \mathbf{m}_b)(\tilde{\mathbf{b}} - \mathbf{m}_b)^T\} \qquad (69)$$

with

$$\tilde{\mathbf{b}} = [\tilde{b}_u^{(0)}, \tilde{b}_v^{(0)}]^T \qquad (70)$$

Since $\mathbf{B}_b^{-1}$ is a positive semi-definite matrix, i.e.,

$$\mathbf{m}_b^T\mathbf{B}_b^{-1}\mathbf{m}_b \geq 0 \ , \qquad (71)$$

we can have

$$0 < f_{\tilde{b}_u^{(0)},\tilde{b}_v^{(0)}|\mathbf{b},\boldsymbol{\rho}}(0,0 \mid \mathbf{b},\boldsymbol{\rho}) \leq \max_{\substack{\rho_{uv} \\ \rho_{uv} \neq \pm 1}} \frac{1}{2\pi\sigma^2\sqrt{1-\rho_{uv}^2}} \, . \qquad (72)$$

With the above results,

$$\min_{\rho_{uv},\rho_{uv} \neq \pm 1}\sqrt{1-\rho_{uv}^2} = \frac{2\sqrt{N-1}}{N} \qquad (73)$$

where [41]

$$\rho_{uv} = \begin{cases} 1 - \frac{2}{N} \quad \text{or} \\ -1 + \frac{2}{N} \end{cases} , \qquad (74)$$

and

$$M\{\rho_{ul}\rho_{vl}\rho_{uv}\} = \sum_{m=1}^{N}\sum_{p=1}^{N}\sum_{q=1}^{N} M\{c_{um}c_{lm}c_{vp}c_{lp}c_{uq}c_{vq}\}$$
$$= \sum_{m=1}^{N}\frac{1}{N^3} = \frac{1}{N^2} \qquad (75)$$

we can derive a range of $a_{1,\text{opt}}$ as follows:

$$\frac{\sum_{u \neq l}^{K} S_u^2(1-2P_{e,u})}{\sum_{u \neq l}^{K} S_u^2 + \frac{1}{\pi\sqrt{N-1}}\sum_{u \neq l}^{K}\sum_{v \neq l,u}^{K} S_u S_v} \leq a_{1,\text{opt}} < 1 - \frac{2\sum_{u \neq l}^{K} S_u^2 P_{e,u})}{\sum_{u \neq l}^{K} S_u^2} \qquad (76)$$

If the power control is perfect, i.e.,

$$S_u = S_v = S \quad \text{and} \quad P_{e,u} = P_e \qquad (77)$$

and P_e is approximated by the BER of high SNR case, i.e., the $Q(\sqrt{\frac{N}{K-1}})$ function [42] and [43], (38) can be rewritten as

$$\frac{1-2Q(\sqrt{\frac{N}{K-1}})}{1+\frac{K-2}{\pi\sqrt{N-1}}} \le a_{1,\text{opt}} < 1-2Q\Big(\sqrt{\frac{N}{K-1}}\Big) \tag{78}$$

It is interesting to see that the lower and upper boundary values can be explicitly calculated from the processing gain N and the number of users K.

5 MMSE GD

5.1 Channel Estimation

It was proved that for ML MIMO channel estimator the optimal pilot matrix minimizing the mean square estimation error is an orthogonal pilot matrix [36], [37]. Under the use of the pilot matrix, i.e.,

$$\mathbf{S}_p\mathbf{S}_p^H = E_P N_P \mathbf{I}_{N_T} \ , \tag{79}$$

where $N_P \ge N_T$ and E_P is the energy of each pilot symbol, the estimated channel matrix can be expressed as [36], [37]

$$\hat{\mathbf{H}} = \mathbf{H} + \Delta\mathbf{H} \ , \tag{80}$$

where

$$\Delta\mathbf{H} = \mathbf{n}\mathbf{S}_p^H (E_P N_P)^{-1} \tag{81}$$

is the channel estimation error matrix, which is correlated with the matrix $\mathbf{H}$ and with entries subjected to Gaussian distribution with zero mean and variance

$$\sigma_{\Delta h}^2 = \sigma_n^2 (E_P N_P)^{-1} \ , \tag{82}$$

which is determined independent of instantaneous channel realization. We can conclude that $\hat{\mathbf{H}}$ is a complex Gaussian matrix with zero mean and covariance matrix

$$Cov[\hat{\mathbf{H}}] = M\{\hat{\mathbf{H}}\hat{\mathbf{H}}^H\} = N_T(\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R}) \ . \tag{83}$$

Let $\mathbf{h}_m, \Delta\mathbf{h}_m$ and $\hat{\mathbf{h}}_m, (m = 1,2,\ldots,N_T)$ denote the m-th column of matrices $\mathbf{H}, \Delta\mathbf{H}$ and $\hat{\mathbf{H}}$, respectively. Then, by the important properties of complex Gaussian random vector [44] and with some manipulations, we obtain

$$M\{\Delta\mathbf{h}_m \mid \hat{\mathbf{h}}_m\} = \sigma_{\Delta h}^2 \hat{\mathbf{h}}_m (\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R})^{-1} \tag{84}$$

and

$$Cov[\Delta\mathbf{h}_m \Delta\mathbf{h}_m^H \mid \hat{\mathbf{h}}_m] = \sigma_{\Delta h}^2 \mathbf{I}_{N_R} - \frac{\sigma_{\Delta h}^4}{\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R}} \ . \tag{85}$$

5.2 Computation of MMSE GD

Let $k_i \in \{1,2,\ldots,N_T\}$ be the index of the i-th detected spatial data stream according to the maximal post-detection signal-to-interference-plus-noise ratio (SINR) ordering rule. Denote μ_{a_j} and $\sigma_{a_j}^2$ as the mean and variance of the signal a_j, respectively, which can be determined by a posteriori symbol probability estimation as in [35]. By performing the soft interference cancellation (SIC) [35] and considering channel estimation error, corresponding interference-cancelled received signal vector $\tilde{\mathbf{y}}_{k_i}$ can be determined in the following form:

$$\tilde{\mathbf{y}}_{k_i} = \mathbf{H}\mathbf{a} - \sum_{j=k_1}^{k_{i-1}} \hat{\mathbf{h}}_j \mu_{a_j} + \mathbf{n} = \sum_{j=k_i}^{k_{N_T}} (\hat{\mathbf{h}}_j - \Delta\mathbf{h}_j) a_j$$
$$+ \sum_{j=k_1}^{k_{i-1}} \hat{\mathbf{h}}_j (a_j - \mu_{a_j}) - \sum_{j=k_1}^{k_{i-1}} \Delta\mathbf{h}_j a_j + \mathbf{n} \ . \tag{86}$$

Then, conditionally on $\hat{\mathbf{H}}$, the MMSE GD is given as [45], [46]

$$\tilde{\mathbf{W}}_i = \frac{M\{2a_{k_i}\tilde{\mathbf{y}}_{k_i}^H \mid \hat{\mathbf{H}}\} - M\{\tilde{\mathbf{y}}_{k_i}\tilde{\mathbf{y}}_{k_i}^H \mid \hat{\mathbf{H}}\} + M\{\mathbf{n}_{1_i}\mathbf{n}_{1_i}^H\}}{M\{\tilde{\mathbf{y}}_{k_i}\tilde{\mathbf{y}}_{k_i}^H \mid \hat{\mathbf{H}}\}} \tag{87}$$

where $\mathbf{n}_1$ is the zero mean Gaussian noise with the following covariance matrix in general case [2], [5]

$$M\{\mathbf{n}_1\mathbf{n}_1^H\} = \sigma_{n_1}^2 \mathbf{I}_{N_R} \ . \tag{88}$$

Thus, according to (86) and (88), we can write

$$M\{\tilde{\mathbf{y}}_{k_i}\tilde{\mathbf{y}}_{k_i}^H \mid \hat{\mathbf{H}}\} = \Big\{\sum_{j=k_i}^{k_{N_T}} [\tilde{\mathbf{h}}_j\tilde{\mathbf{h}}_j^H - \tilde{\mathbf{h}}_j M\{\Delta\mathbf{h}_j^H \mid \hat{\mathbf{H}}\}$$
$$- M\{\Delta\mathbf{h}_j \mid \hat{\mathbf{H}}\}\tilde{\mathbf{h}}_j^H] + \sum_{j=1}^{k_{N_T}} M\{\Delta\mathbf{h}_j\Delta\mathbf{h}_j^H \mid \hat{\mathbf{H}}\}\Big\} E_b$$
$$+ \sum_{j=k_1}^{k_{i-1}} [\hat{\mathbf{h}}_j\hat{\mathbf{h}}_j^H - \hat{\mathbf{h}}_j M\{\Delta\mathbf{h}_j^H \mid \hat{\mathbf{H}}\} - M\{\Delta\mathbf{h}_j \mid \hat{\mathbf{H}}\}\hat{\mathbf{h}}_j^H]\sigma_{a_j}^2$$
$$+ (\sigma_{n_1}^2 - \sigma_n^2)\mathbf{I}_{N_R} \ . \tag{89}$$

Based on results of Section *5.1*, it is evidently that $\Delta\mathbf{h}_j$ is only correlated with $\hat{\mathbf{h}}_j$. Then, we have

$$M\{\Delta\mathbf{h}_j\Delta\mathbf{h}_j^H \mid \hat{\mathbf{H}}\} = M\{\Delta\mathbf{h}_j\Delta\mathbf{h}_j^H \mid \hat{\mathbf{h}}_j\} \ . \tag{90}$$

From the basic relationship between the autocorrelation and covariance functions, we have

$$M\{\Delta\mathbf{h}_j\Delta\mathbf{h}_j^H \mid \hat{\mathbf{h}}_j\} = Cov\{\Delta\mathbf{h}_j\Delta\mathbf{h}_j^H \mid \hat{\mathbf{h}}_j\}$$
$$+ M\{\Delta\mathbf{h}_j \mid \hat{\mathbf{h}}_j\} M\{\Delta\mathbf{h}_j^H \mid \hat{\mathbf{h}}_j\} \ . \tag{91}$$

Substituting (84) and (85) into (91), we can write

$$M\{\Delta\mathbf{h}_j \Delta\mathbf{h}_j^H \mid \hat{\mathbf{h}}_j\} = \sigma_{\Delta h}^2 \mathbf{I}_{N_R} - \sigma_{\Delta h}^4 (\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R})^{-1} + \sigma_{\Delta h}^4 (\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R})^{-1} \mathbf{h}_j \mathbf{h}_j^H (\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R})^{-1} \ . \tag{92}$$

Introduce the following notations

$$\mathbf{\Lambda} = (\mathbf{R}_\mathbf{r} + \sigma_{\Delta h}^2 \mathbf{I}_{N_R})^{-1} \ , \tag{93}$$

$$\mathbf{\Xi} = \mathbf{I}_{N_R} - \sigma_{\Delta h}^2 \mathbf{\Lambda} \ , \tag{94}$$

$$\mathbf{R}_{aa} = diag\{\sigma_{a_{k_1}}^2, \sigma_{a_{k_2}}^2, \ldots, \sigma_{a_{k_{i-1}}}^2\} \ . \tag{95}$$

Substituting (84) and (92) into (89) and taking into consideration (93)-(95), we have

$$M\{\tilde{\mathbf{y}}_{k_i} \tilde{\mathbf{y}}_{k_i}^H\} = E_b \mathbf{\Xi} \hat{\mathbf{H}}_{k_i:k_{N_T}} \hat{\mathbf{H}}_{k_i:k_{N_T}}^H \mathbf{\Xi} + E_b \sigma_{\Delta h}^4 \mathbf{\Lambda} \hat{\mathbf{H}}_{k_1:k_{i-1}} \times \hat{\mathbf{H}}_{k_1:k_{i-1}}^H \mathbf{\Lambda} + (\hat{\mathbf{H}}_{k_1:k_{i-1}} \mathbf{R}_{aa} \hat{\mathbf{H}}_{k_1:k_{i-1}}^H + N_T E_b \sigma_{\Delta h}^2) \mathbf{\Xi} - \sigma_{\Delta h}^2 \mathbf{\Lambda} \hat{\mathbf{H}}_{k_1:k_{i-1}} \mathbf{R}_{aa} \hat{\mathbf{H}}_{k_1:k_{i-1}}^H + (\sigma_{n_1}^2 - \sigma_n^2) \mathbf{I}_{N_R} \ , \tag{96}$$

where the notation $\mathbf{H}_{n:m}$ denotes the submatrix containing the n-th to m-th columns of the matrix $\mathbf{H}$. Based on similar manipulations, we can write

$$M\{a_{k_i} \tilde{\mathbf{y}}_{k_i}^H \mid \hat{\mathbf{H}}\} = E_b \hat{\mathbf{h}}_{k_i}^H \mathbf{\Xi} \ . \tag{97}$$

Combining (96) and (97), we obtain the MMSE GD $\tilde{\mathbf{W}}_i$, conditionally on $\hat{\mathbf{H}}$.

5.3 Computation of LLR

By applying $\tilde{\mathbf{W}}_i$ to $\tilde{\mathbf{y}}_{k_i}$, we have the process at the output of the MMSE GD [45], [46]

$$\tilde{z}_{k_i} = \tilde{\mathbf{W}}_i \tilde{\mathbf{y}}_{k_i} \ . \tag{98}$$

According to the Gaussian approximation of the MMSE GD output process, we can write

$$\tilde{z}_{k_i} \approx \tilde{\mu}_{k_i} a_{k_i} + \tilde{\eta}_{k_i} \ , \tag{99}$$

where

$$\tilde{\mu}_{k_i} = M\{\tilde{\mathbf{W}}_i (\hat{\mathbf{h}}_{k_i} - \Delta\mathbf{h}_{k_i}) \mid \mathbf{H}\} = \tilde{\mathbf{W}}_i \mathbf{\Xi} \hat{\mathbf{h}}_{k_i} \tag{100}$$

and $\tilde{\eta}_{k_i}$ is a zero-mean complex Gaussian random variable with variance

$$\sigma_{\tilde{\eta}_{k_i}}^2 = E_b (\tilde{\mu}_{k_i} - \tilde{\mu}_{k_i}^2) \ . \tag{101}$$

Therefore, the LLR value of the coded bit $\mathbf{g}_{k_i}^\lambda$ can be approximated as [33], [34]

$$\mathscr{L}(\mathbf{g}_{k_i}^\lambda) \approx \frac{1}{\sigma_{\tilde{\eta}_{k_i}}^2} (\min_{\alpha_i \in \mathscr{F}_\lambda^0} |z_i - \mu_i \alpha_i|^2 - \min_{\alpha_i \in \mathscr{F}_\lambda^1} |z_i - \mu_i \alpha_i|^2) \tag{102}$$

where $\mathscr{F}_\lambda^0$ and $\mathscr{F}_\lambda^1$ denote the modulation constellation symbols subset of $\mathscr{F}$ whose λ-th bit equals 0 and 1, respectively.

5.4 Remarks

When the channel estimation error is neglected, i.e., $\sigma_{\Delta h}^2 = 0$ in (93), (94) and (96), the MMSE GD given by (87) reduces to that of the modified soft-output MMSE GD, in which only decision error propagation is considered [46]. On the other hand, if $\mathbf{R}_\mathbf{r} = \mathbf{I}_{N_R}$ and no residual interference cancellation error is assumed the MMSE GD of (87) reduces to that of [45]. For the sake of simplicity, we call this detector as the conventional soft-output MMSE GD hereafter if this detector is applied in channel coded MIMO system. Meanwhile, if both decision error propagation and channel estimation error are neglected, the MMSE GD of (87) reduces to that of the conventional MMSE GD of [47].

6 Simulation Results

6.1 Selection/Maximal-Ratio Combining

In this subsection we discuss some examples of the GD performance under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes and compare with the conventional hybrid selection/maximal-ratio combining receiver.

The average SER of coherent BPSK and 8-PAM signals under processing by the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes as a function of average SNR per symbol per diversity branch for various values of $2L$ and $2N = 8$ is presented in Fig.2. It is seen that the GD performance with quadrature subbranch hybrid selection/ maximal -ratio combining and hybrid selection/maximal-ratio combining schemes with $(L, N) = (3,4)$ achieves virtually the same performance as the GD with traditional maximal-ratio combining, and that the performance with $(L, N) = (2,4)$ is typically less than 0.5 dB in SNR poorer than the GD with traditional maximal-ratio combining in [19]. Also, a comparison with the traditional hybrid selection/maximal-ratio combining receiver [6] is made. Advantage of GD employment is evident.

Average SER of coherent BPSK and 8-PAM signals under processing by the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes as a function of average SNR per symbol per diversity branch for various values of $2N$ with $2L = 4$ is shown in Fig.3. We note the substantial benefits of increasing the number of diversity branches N for fixed L. Comparison with the traditional hybrid selection/maximal-ratio combining receiver [6] is made. Advantage of GD employment is evident.

Comparative analysis of the average BER as a function of the average SNR per bit per diversity branch of coherent BPSK signals under the use of the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection /maximal-ratio combining schemes and the GD with traditional hybrid selection/maximal-ratio combining scheme for various values of L with $N = 4$ is presented in Fig.4.

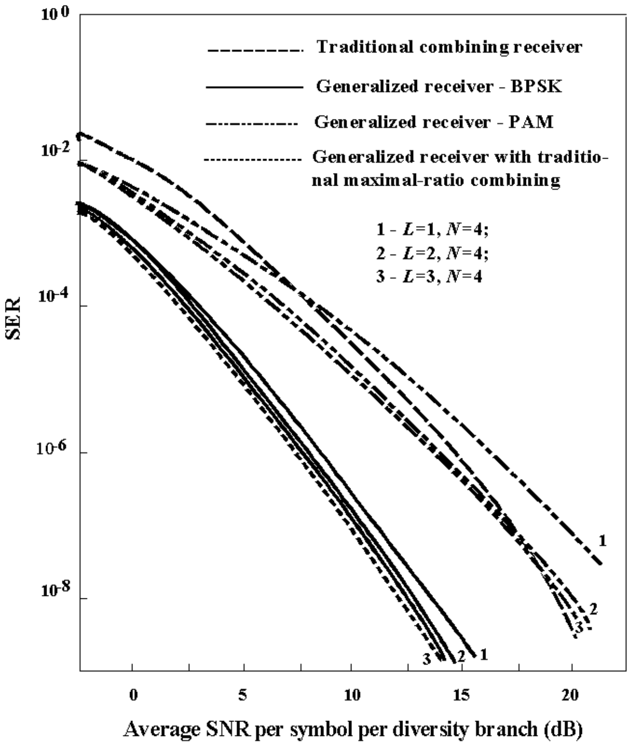

Figure 2. Average SER of coherent BPSK and 8-PAM for the GD under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes versus the average SNR per symbol per diversity for various values of $2L$ with $2N = 8$.

To achieve the same value of average SNR per bit per diversity branch, we should choose $2L$ quadrature branches for the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes and L diversity branches for the GD with traditional hybrid selection/maximal-ratio combining scheme.

Figure 4 shows that the performance of the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combi

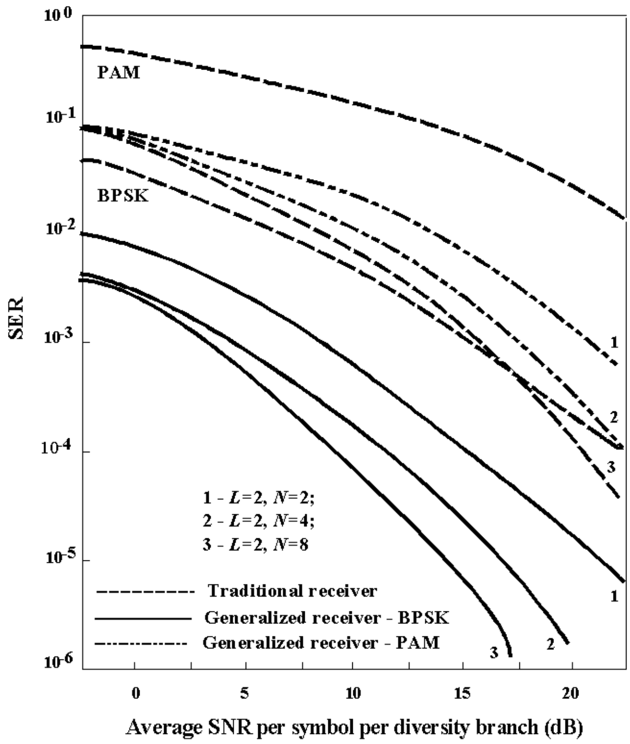

Figure 3. Average SER of coherent BPSK and 8-PAM for the GD under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes versus the average SNR per symbol per diversity for various values of $2N$ with $2L = 4$.

ning schemes is much better than that of the GD with traditional hybrid selection/maximal-ratio combining scheme, about 0.5 dB to 1.2 dB when L is less than one half N. This difference decreases with increasing L. This is expected because when $L = N$ we obtain the same performance. Some discussion of the increases in GD complexity and power consumption is in order.

We first note that the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes requires the same number of antennas as the GD with traditional hybrid selection/maximal-ratio combining scheme. On the other hand, the former requires twice as many comparators as the latter, to select the best signals for further processing. However, the GD designs that process of the quadrature signal components will require $2L$ receiver chains for either the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes or the GD with traditional hybrid selection/maximal-ratio combining scheme. Such receiver designs will use only little additional power, as the

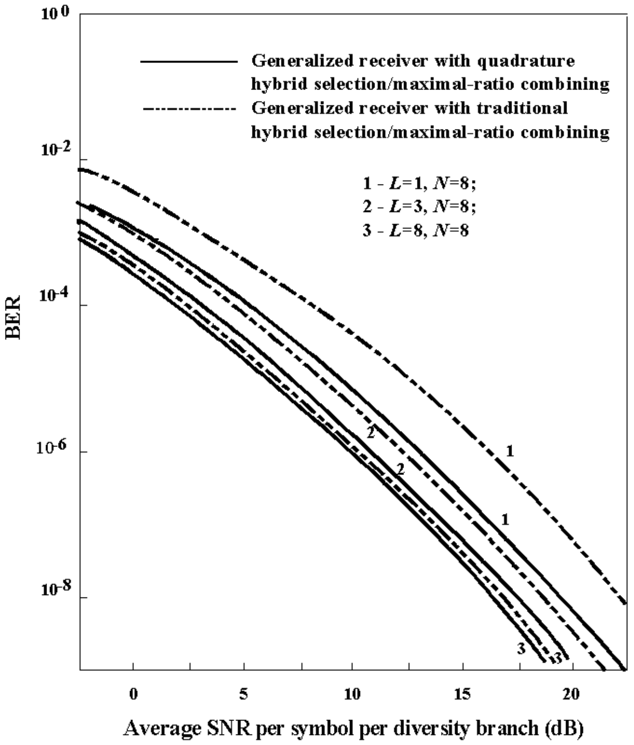

Figure 4. Comparison of the average BER of coherent BPSK and 8-PAM for the GD under quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes for various values of $2L$ with $N = 8$.

GD chains consume much more power than the comparators.

On the other hand, the GD designs that implement co-phasing of the branch signals without splitting the branch signals into the quadrature components will require L receiver chains for the GD with traditional hybrid selection/maximal-ratio combining scheme and $2L$ receiver chains for the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes, with corresponding hardware and power consumption increases.

6.2 Synchronous DS-CDMA System

To demonstrate the usefulness of the range of the optimal partial cancellation factor given by (78), we performed a number of simulations for a synchronous DS-CDMA system with perfect power control. In the simulations, the random spreading codes with length $N = 64$ were used for each user and the number of users was $K = 40$ [48].

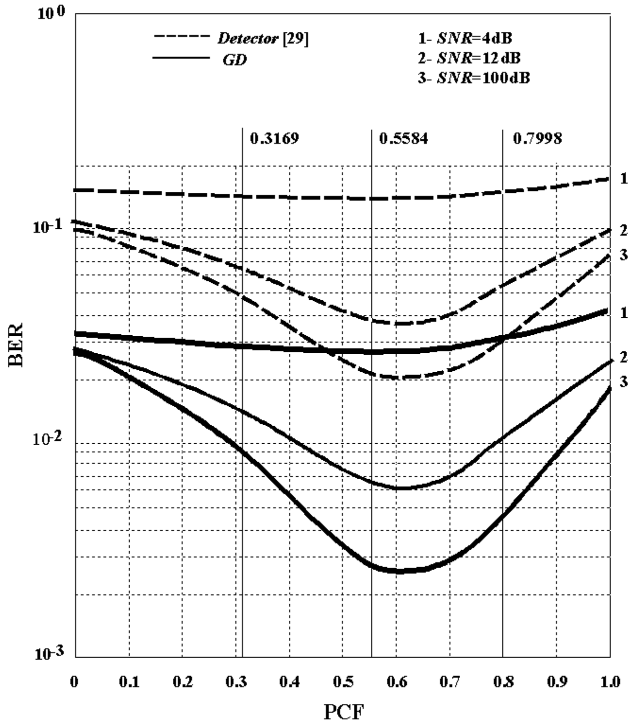

Figure 5. The BER performance of the single-stage GD based on the partial parallel interference cancellation with hard decisions for different SNRs and partial cancellation factors.

Figure 5 shows the BER performance of the single -stage hard-decision GD based on the partial parallel interference cancellation for different magnitudes of SNR and various values of partial cancellation factors, where the optimal partial cancellation factor for the first stage lies between 0.3169 (lower boundary) and 0.7998 (upper boundary). It can be seen that, for all the SNR cases, the GD based on the partial parallel interference cancellation using the average of the lower and upper boundary values, i.e., 0.5584, as the partial cancellation factor, has close BER performance to that using the optimal partial cancellation factor Also, the comparative results under GD employment in DS-CDMA systems with the detector discussed in [29] are presented. These results show us a great superiority of the GD employment in comparison with detector investigated in [29].

Figure 6 shows the BER performance at each stage for the three-stage GD based on the partial parallel interference cancellation using different partial cancellation factors at the first stage, i.e., the average value and an arbitrary value. The partial cancellation factors for these two three-stages cases are

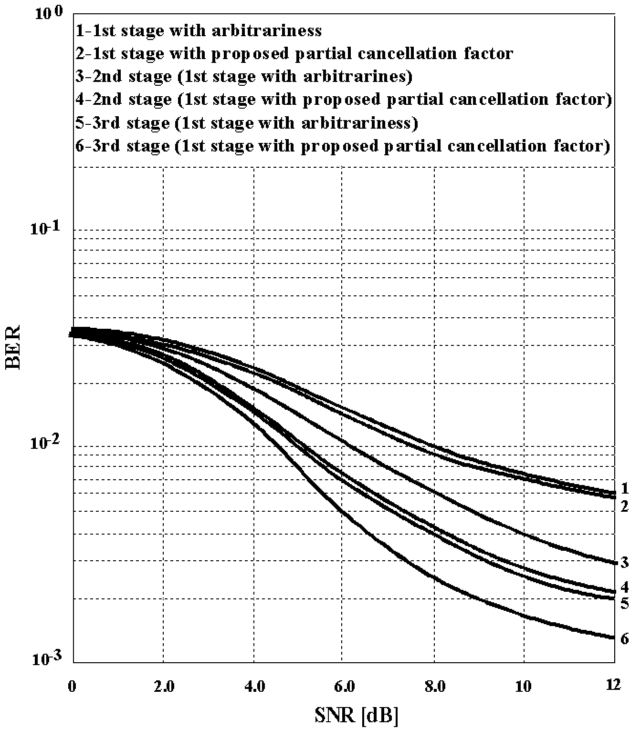

Figure 6. The BER performance at each stage for three-stage GD based on he partial parallel interference cancellation with hard decisions for different partial cancellation factors at the first stage, i.e., the average value and an arbitrary value.

$$(a_1, a_2, a_3) = \begin{cases} (0.5584, 0.8, 0.9) \\ \text{and} \\ (0.7, 0.8, 0.9) \end{cases} \quad (103)$$

respectively.

The results demonstrate that the BER performances of the GD employed by DS-CDMA systems of the cases using the proposed in [29] partial cancellation factor at the first stage are better than the BER performances of the GD implemented in DS-CDMA system using an arbitrary partial cancellation factor at the first stage. Furthermore, the BER performance of the GD at the second stage for the case using the proposed in [29] the partial cancellation factor at the first stage achieves the BER performance of the GD comparable to that of the three-stage GD based on partial parallel interference cancellation using an arbitrary partial cancellation factor at the first stage.

6.3 MIMO System

We choose the 0.5 rate Low Density Parity Check (LDPC) code with a block length of 64800 bits, which is also adopted by DVB-S.2 standard [49]. QPSK modulation with Gray mapping is adopted in $N_T = N_R = 4$ MIMO system. Meanwhile, we set $\sigma^2_{\Delta h} = 1$, $N_P = N_T$, and $E_P = E_S$. The channel is generated with coherence time of $N_P + N_D = 85$ MIMO vector symbol intervals, and then a LDPC codeword is transmitted via 100 channel coherent time intervals for QPSK modulation.

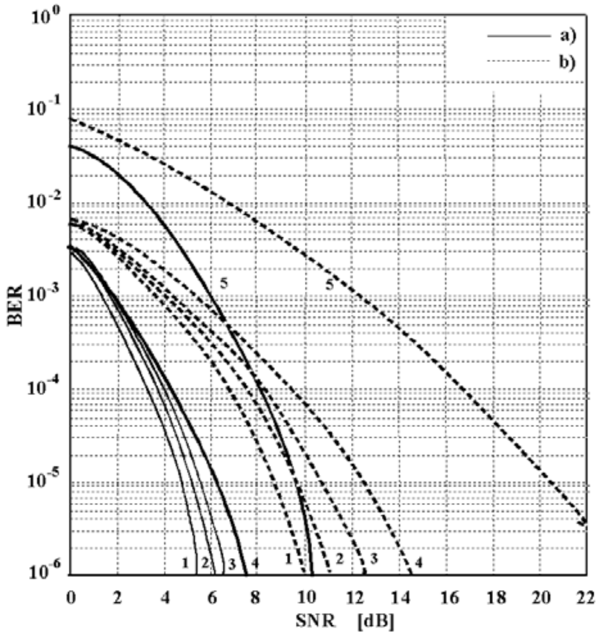

Figure 7. BER performance of different detectors under a)spatially independent MIMO channel and b)receiver spatially correlated MIMO channel: 1–MMSE GD; 2–MMSE GD [46]; 3–MMSE GD [45]; 4–MMSE GD [47]; 5–MMSE V-BLAST detector [39].

For GD spatially correlated MIMO channel the GD array correlation matrix $\mathbf{R}_\mathbf{r}$ with the following elements is adopted [50]

$$\begin{cases} \mathbf{R}_\mathbf{r}(n,n) = 1, \mathbf{R}_\mathbf{r}(m,n) = \mathbf{R}^*_\mathbf{r}(m,n), m,n = 1,2,3,4 \\ \mathbf{R}_\mathbf{r}(1,2) = \mathbf{R}_\mathbf{r}(2,3) = \mathbf{R}_\mathbf{r}(3,4) = 0.4290 + 0.7766j \\ \mathbf{R}_\mathbf{r}(1,3) = \mathbf{R}_\mathbf{r}(2,4) = -0.3642 + 0.5490j \\ \mathbf{R}_\mathbf{r}(1,4) = -0.4527 - 0.0015j \end{cases} \quad (104)$$

The performance comparison, in terms of BER, of the proposed soft-output MMSE GD (the curve 1), the modified soft-output MMSE GD [46] (the curve 2), the conventional soft-output MMSE GD [45] (the curve 3), and the conventional MMSE GD [47] (the curve 4) is presented in Fig.7 for spatially independent MIMO channel and receiver spatially correlated MIMO channel. Also, a comparison with the soft-output MMSE V-BLAST detector discussed in [39] is made.

The proposed MMSE GD outperforms all the existing schemes with considerable gain, especially for receiver correlation MIMO channel scenario. The underlying reason of this improvement is that the MMSE GD, by taking channel estimation error, decision error propagation and channel correlation into account, can output more reliable LLR to channel decoder. As channel estimation error is the dominant factor influencing the system performance under the lower SNR region, it can observed that the BER of the conventional soft-output MMSE GD [45] is slightly better than that of the modified soft-output MMSE GD in the case of spatially independent MIMO channel.

7 Conclusions

In this paper, the performance of the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes for 1-D signal modulations in Rayleigh fading is investigated. The SER of M-ary pulse amplitude modulation, including coherent BPSK modulation, is derived.

Results show that the GD with quadrature subbranch hybrid selection/maximal-ratio combining and hybrid selection/maximal-ratio combining schemes performs substantially better than the GD with traditional hybrid selection/ maximal-ratio combining scheme, particularly when L is smaller than one half N, and much better than the traditional hybrid selection/maximal-ratio combining receiver.

We have also derived a range of the optimal partial cancellation factor for the GD first stage based on the partial parallel interference cancellation which is employed by DS-CDMA system, with hard decisions in AWGN environment. Computer simulation results shown that the BER performance of the GD employed by DS-CDMA system of the case using the average of the lower and upper boundary values is close to the BER performance of the GD of the case using the real optimal partial cancellation factor, whether the SNR is high or low.

It has also been shown that the GD employment in DS-CDMA system allows us to observe a great superiority over the detector discussed in [29]. The procedure discussed in [29] is also acceptable under the GD employment by DS-CDMA systems. It has also been demonstrated that the two-stage GD based on the partial parallel interference cancellation using the proposed in [6] the partial cancellation factor at the first stage achieves the GD BER performance comparable to that of the three-stage GD based on the partial parallel interference cancellation using an arbitrary partial cancellation factor at the first stage.

This means that under the same BER performance, the number of stages (or complexity) required for the multistage GD based on the partial parallel interference cancellation can be reduced when the proposed in [29] partial cancellation factor is used at the first stage. As well as AWGN environments, it can be shown that the proposed in [29] partial cancellation factor selection approach is applicable to multipath fading cases under GD employment in DS-CDMA systems even if non-perfect power control is assumed.

The proposed MMSE GD outperforms all the existing schemes with considerable gain especially for receiver correlation MIMO channel scenario. The underlying reason of this improvement is that the MMSE GD, by taking channel estimation error, decision error propagation, and channel correlation into account, can output more reliable LLR to channel decoder. As channel estimation error is the dominant factor influencing the system performance under lower SNR region, it can observed that the BER of the conventional soft-output MMSE GD [45] is slightly better than that of the modified soft-output MMSE GD in the case of spatially independent MIMO channel.

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